

**Automated Incorporation Of Stability
Criteria In The Design Of Steel
Frames Members**

BY

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**Submitted in partial fulfillment of the requirements for
the degree of master of science in civil engineering.**

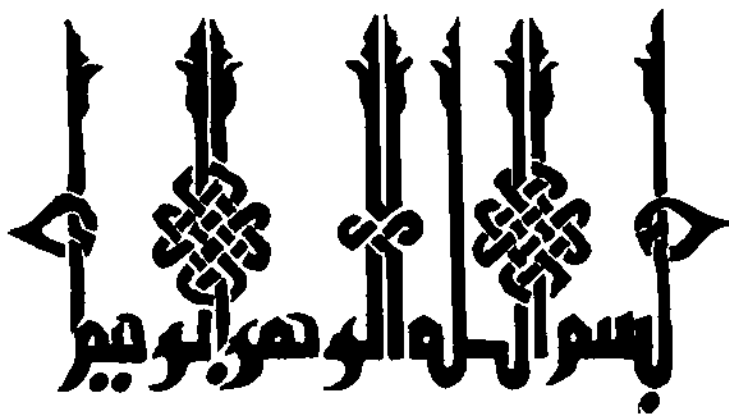
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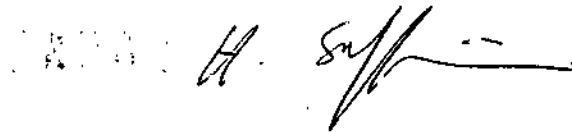
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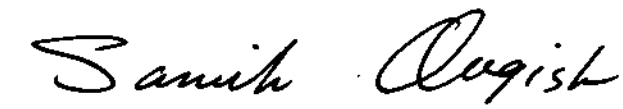
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الإهداء

الى روح أبي

الى أمي

الى أخي بركات

الى اخوتي واخواتي جميعاً

أهدي ثمرة جهدي هذا مع المحبة والتقدير والاحترام

ABSTRACT

A computer program that studies over all stability of framed structures has been developed, with the purpose of comparison between finite element approach and stability function approach on the one hand, and alignment charts on the other. Influence of errors in effective length as computed using alignment charts on design as determined by the criteria of allowable stress is also studied.

Effect of primary bending moment and lateral loads on the stability of frames as determined by the elastic critical load and effective length factors are also studied.

Finally another computer program for elastic-plastic analysis is developed to determine the failure load factor for the frame and its relation with the elastic critical load factor as calculated by the first program.

ACKNOWLEDGMENTS

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LIST OF TABLES

Tables (4.1-4.8) Comparison of effective factors as calculated by using the alignment charts, computer, using the finite element approach, and computer using the stability functions for sway and nonsway frames.	(65-67)
Table-4.9 Critical load factors	(68)
Table-4.10 Comparison of CPU time	(68)
Table- 4.11 Collapse load factors calculated using PLAS for some selected frames	(73)
Table-5.1 Effect of primary bending moment on λ_c .	(84)
Table-5.2 Effect of lateral loads on λ_c .	(84)
Table- 5.3 Effect of lateral loads on effective load factors	(84)
Table- 5.4 - 5.6 Errors resulting from using alignment charts to calculate effective length and stress ratio.	(87)
Table- 5.7 Comparison between failure load factors calculated using PLAS and failure load factors from Marchent-Rankine Formula.	(94)

TABLES OF CONTENTS

ABSTRACT.....	(VI)
ACKNOWLEDGEMENT.....	(VII)
LIST OF TABLES.....	(VIII)
CHAPTER ONE:- REVIEW OF THE STATE OF THE ART	1
1.1 General	2
1.2 Previous research	4
1.3 Alignment charts	8
CHAPTER TWO:- THEORETICAL BACKGROUND	
2.1 The concept of elastic stability	11
2.2 Effect of axial forces	13
2.2.1 Stiffness of a member subjected to axial force..	14
2.2.1.1 Other forms of the stability functions..	18
2.2.2 Finite element method	20
2.3 Implementation	23
2.3.1 Formulation of the global stiffness matrix.....	24
2.3.2 A solution method for eigenvalue problem.....	25
2.3.2.1 Solution algorithm for determinant search	
method	26
2.3.3 General algorithm for elastic stability analysis	
	28
2.4 Elastic-Plastic analysis.....	28
2.4.1 Modification of members stiffness due to	

development of a plastic hinge.....	30
2.4.2 Effect of the axial force on the plastic moment capacity.....	31
2.4.3 Calculation of failure loads	32
CHAPTER THREE:- COMPUTER PROGRAMS	34
3.1 Elastic stability analysis program for plane frames...	35
3.1.1 Program capacity	36
3.1.2 Internal organization of ESAP	37
3.2 Elastic-plastic analysis program for plane frames.....	46
CHAPTER FOUR:- EXAMPLES.....	52
4.1 Results of the elastic stability analysis	53
4.1.1 Finite element versus stability functions	63
4.1.2 Comparison with alignment charts	69
4.2 Elastic-plastic analysis	72
CHAPTER FIVE:- DISCUSSION	78
5.1 Elastic stability analysis.....	79
5.1.1 Effective length factor -definition and use according to AISC code	80
5.1.2 Effective length factor as calculated by ESAP...	81
5.1.3 Discussion of k-factor results.....	86
5.2 Elastic-Plastic analysis	92
CHAPTER SIX:- CONCLUSION	95
APPENDICES	
APPENDIX I DATA ENTRY	99
REFERENCES	104

CHAPTER ONE
REVIEW OF THE STATE OF THE ART

CHAPTER I

REVIEW OF THE STATE OF THE ART

1.1 GENERAL

Although the use of columns goes back to the dawn of history, it was not until 1789 that a paper was published concerning the buckling of columns by Leonhard Euler [1], a Swiss mathematician, who was probably the first person to realize the significance of buckling.

The derivation of Euler formula, the most famous of all column expression, marked the real beginning of theoretical and experimental investigation of columns.

Nowadays it is accepted that only very short steel columns can be loaded to their yield stress. The usual situation is that buckling, or sudden bending as a result of instability, occurs prior to developing the full material strength of the member.

A sound knowledge of member stability is hence,

necessary for the design of structural steel-compression elements.

Formulas for column capacity with ideal end conditions are readily available in the literature. These conditions, however, are not encountered in actual practice.

Most practical columns are found as a part of engineering structures, and not as isolated members. Their ends are not clearly hinged, fixed, or free. Instead, these members are usually connected to other members, and their ends are elastically restrained by the members to which they are attached. It is thus desirable that we consider the behavior of framed columns.

In frames, no single member can buckle without being affected by other members, that is to say that the elastic restraint at the end of a given compression member depends not only on the members framing directly into it, but on all members in the frame. So to obtain the critical load of one compression member in a frame it is necessary to find the critical load for the entire frame acting as single unit.

To simplify design, the concept of the

effective column length is used. This length is a modified length of the member incorporating some information on the global behavior of the structure particularly the effect of adjacent elements.

1.2 PREVIOUS RESEARCH

Many papers on the stability of frames have been published, but the practical use of these methods is quite limited in practice, this is because codes of practice still endorse the use of simple and direct methods to get what is called effective length factor.

Early in 1935 James [2] introduced the concept of stability functions to include the effect of the axial forces in moment distribution factors used in the analysis of rigid frames. Lundquist later [3] extended James' work to allow for determination of the elastic critical loads on frames. Actually Lundquists' work is considered to be the foundation stone in the study of stability of rigid frames.

In 1948 Winter at el. [4] presented a relaxation technique employed to evaluate the critical load of frames, essentially, the process is one of the moment distribution where a suitable disturbance is applied to the frame which

may be a unit rotation of a single joint, and a moment distribution process is carried out, the moment which remains at the joint is a measure of the frame stiffness with respect to that joint, the process is repeated with different load parameter, and finally a curve is drawn that describes the relationship between stiffness and load parameter. The critical load is then defined as the zero stiffness load parameter.

In 1955 Bolton [5,6,7] used Winter relaxation technique but on a suitable substitute frame rather than the actual one so he was able to get a quick approximation to the critical load factor of practical rigid frames.

In 1954 Marchant [8] suggested an interaction formula to calculate the failure load factor of frames (λ_f) by using the elastic critical load factor λ_c and the plastic collapse load factor λ_p .

In 1955 Marchant et al. [9,10,11] started publishing a series of papers describing methods that approximately estimate the critical loads of multi-bay, multi-storey frames by reducing them to equivalent single bay frames. The methods described are dependent on the use of the stability functions tabulated by Livesley and Chandler [12]. these are iterative

methods in which the solution is reached when the determinant of the stiffness matrix reaches zero.

In 1967 Stevens [13] presented a procedure for nonbraced multi-storey, multi-bay frames based on the energy method. The procedure requires elastic analysis of the frame under the action of lateral loads, and the resulting deflection is used to estimate the internal strain energy in the structures in its buckled mode, the procedure avoids the use of stability functions but it is difficult and potentially unsafe.

In 1974 Wood [14] modified the famous Rankin-Marchent formula, and presented a new technique of stiffness distribution, and used it to solve the frame instability problem by reducing the actual frame to an equivalent beam/column system before applying stiffness distribution technique. Woods also presented a comprehensive design chart for effective lengths of columns with any local degree of end restraint.

In 1975 Horne [15] presented a method of estimating the elastic critical load of nonbraced multi-storey rigid frames. The method is based on Stevens' procedure and, requires the performance of a linear elastic analysis for a

factor. It is recommended for use in the present AISC and CAN3-M84 specifications.

The alignment charts were originally developed by Julian and Lawrence and were presented in details by Kavanagh [26]. A quick look at the assumptions used in the charts' development shows that its an ideal condition tool. One modification to the alignment charts came from Chu et al [27] as a modification allowing their use for unsymmetrical frames. Another modification, or extension, to the charts was proposed by Yara [28] to account for inelastic column buckling.

Kuhn and Lundgren [29] examined the accuracy of the alignment charts and found that:

- 1- For non-symmetrical frames and loading the effective length factors can be considerably in error.
- 2- The greatest error occurs generally in the more slender columns.

Bridge and Fraser [30] expanded the alignment charts to account for the presence of axial force in the restraining member by using of an iterative procedure based on linearized stability functions.

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Duan and Chen [31,32] modified the alignment charts by considering the effect of far end condition of the column above and below the column under consideration.

CHAPTER TWO
THEORETICAL BACKGROUND

CHAPTER II

THEORETICAL BACKGROUND

2.1 THE CONCEPT OF ELASTIC STABILITY :

A system is said to be stable if it can sustain a small disturbance from the equilibrium position with small response, while if the response is large, the system is then said to be unstable.

The system under consideration in this thesis is a plane frame, and the forces applied to the frame at the stage of the commencement of instability are called the elastic critical loads for that frame. It is elastic as it is calculated with the assumption that the material remains elastic irrespective of the stress level. Actually this assumption is not realistic but it is useful when stability is to be considered in isolation from plastic behavior.

The equations of equilibrium of a discretized structural system that is subjected to loads R and undergoes

displacements U in the corresponding degrees of freedom may be written as follows :-

$$KU = R \quad (2.1)$$

where

K is termed the stiffness matrix of the structure at a specific level of loading.

If almost zero distributing forces are required to cause large displacement, which occurs in instability state then :-

$$KU = 0 \quad (2.2)$$

For nontrivial solution, $U \neq 0$

$$| K | = 0$$

This means that the stiffness matrix $[K]$ is singular. The singularity of K is equivalent to the structure stiffness being zero.

2.2 EFFECT OF AXIAL FORCES :

In a member subject to axial forces two types of effects due to these axial forces may be important, these are:

1. Effect of axial deformation (i.e change in member length). This effect can easily be included in the analysis by the direct stiffness method.
2. Change of the stiffness of the member due to secondary bending caused by the axial force or what is called the beam-column effect.

2.2.1 STIFFNESS OF A MEMBER SUBJECTED TO AXIAL FORCE: EXACT SOLUTION

Consider the beam column element shown in Figure (2.1)

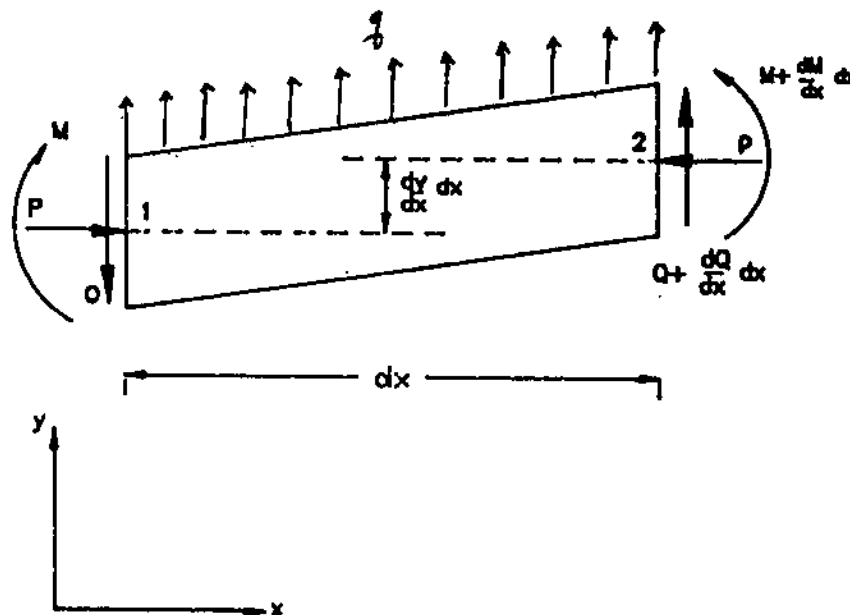


Fig.(2.1) An infinitesimal segment of a beam-column element.

Summation of forces in Y-direction

$$\frac{dQ}{dx} + q = 0 \quad (2.3)$$

Summation of moment at end 1 or 2 gives

$$\frac{dM}{dx} + Q + P \frac{dy}{dx} = 0 \quad (2.4)$$

Differentiating with respect to x while assuming P and EI constant, and substituting $-q$ for $\frac{dQ}{dx}$ gives

$$M'' + Py'' = q \quad (2.5)$$

Knowing that $M = EIy''$ equation (2.5) becomes

$$\frac{d^4y}{dx^2} + \frac{P}{EI} \frac{d^2y}{dx^2} = \frac{q}{EI} \quad (2.6)$$

The general solution of this equation when $q = 0$ is

$$y = A_1 \sin 2\alpha \frac{x}{L} + A_2 \cos 2\alpha \frac{x}{L} + A_3 x + A_4 \quad (2.7)$$

Where :

$$\alpha = \frac{L}{2} \sqrt{P/EI}$$

and $A_1, A_2, A_3,$ and A_4 are integration constants to be determined from the boundary conditions.

Equation 2.7 will be used to derive the stiffness matrix of the member shown in Fig. (2.2).

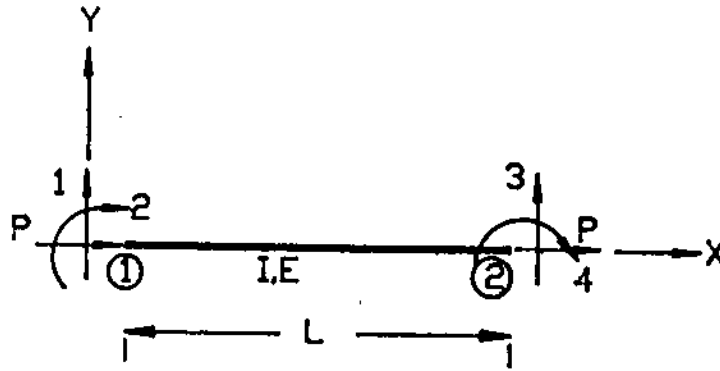


FIG.(2-2) Coordinate system of the member .

The forces at the ends of the element are given by:-

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = \begin{Bmatrix} (-V)_{x=0} \\ (M)_{x=0} \\ (V)_{x=l} \\ (-M)_{x=l} \end{Bmatrix} = EI \begin{Bmatrix} \left(\frac{d^3y}{dx^3} + \frac{(2\alpha)^2}{l^2} \frac{dy}{dx} \right)_{x=0} \\ \left(\frac{-d^2y}{dx^2} \right)_{x=0} \\ \left(\frac{-d^3y}{dx^3} - \frac{(2\alpha)^2}{l^2} \frac{dy}{dx} \right)_{x=l} \\ \left(\frac{d^2y}{dx^2} \right)_{x=l} \end{Bmatrix} \quad (2.8)$$

The displacements at the ends of the member are given by :

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} (y)_{x=0} \\ \left(\frac{dy}{dx} \right)_{x=0} \\ (y)_{x=l} \\ \left(\frac{dy}{dx} \right)_{x=l} \end{Bmatrix} \quad (2.9)$$

Using equations (2.7), (2.8) and (2.9) the following stiffness matrix for a prismatic element can be derived [33.]

$$K = \frac{EI}{2 - 2m - 2\alpha n} \begin{vmatrix} \frac{(2\alpha)^2 n}{l^3} & & & & & \\ \frac{(2\alpha)^2(1-m)}{l^3} & \frac{2\alpha(n-2\alpha m)}{l} & & & & \\ \frac{(-2\alpha)^2 n}{l^3} & \frac{-(2\alpha)^2(1-m)}{l^3} & \frac{(2\alpha)^2 h}{l^3} & & & \\ \frac{(2\alpha)^2(1-m)}{l^3} & \frac{2\alpha(2\alpha-n)}{l} & \frac{-(2\alpha)^2(1-m)}{l^3} & \frac{(2\alpha)(n-2\alpha m)}{l} & & \\ & & & & & \end{vmatrix}$$

----- (2.10)

Where $n = \sin 2\alpha$, and $m = \cos 2\alpha$

$K_{2,2}$ is the moment at end one when a unit rotation is introduced there, while keeping U_2, U_3 and $U_4 = 0$, and $K_{4,2}$ is the carryover moment to end two.

$K_{2,2}$ is known as rotational stiffness S and $K_{4,2}$ is the carryover moment t , so the carryover factor $C = C_{12} = C_{21}$ is

$$C = \frac{2\alpha - \sin 2\alpha}{\sin 2\alpha - 2\alpha \cos 2\alpha} \tag{2.11}$$

$$S = \frac{2\alpha(\sin 2\alpha - 2\alpha \cos 2\alpha)}{2 - 2\cos 2\alpha - 2\alpha \sin 2\alpha} \tag{2.12}$$

or

$$S = \frac{\alpha(1 - 2\alpha \cot 2\alpha)}{\tan \alpha - \alpha} \quad (2.12')$$

Where S and C are known as the stability functions and they tend to 4 and, 0.5 respectively when $P = 0$

Using equations 2.11, and 2.12' the stiffness matrix K (eq. 2.10) can be reduced to

$$K = EI \begin{bmatrix} \frac{2S(1+C) - \pi^2 \rho}{l^2} & & & \\ \frac{S(1+C)}{l^2} & \frac{S}{l} & & \\ -\frac{2S(1+C) - \pi^2 \rho}{l^2} & -\frac{S(1+C)}{l} & \frac{2S(1+C) - \pi^2 \rho}{l^2} & \\ \frac{S(1+C)}{l^2} & \frac{SC}{l} & -\frac{S(1+C)}{l^2} & \frac{S}{l} \end{bmatrix} \quad (2.13)$$

Where $\rho = \sqrt{P/PE}$, and $2\alpha = \pi \sqrt{\rho}$

2.2.1.1 OTHER FORMS OF THE STABILITY FUNCTIONS

For the purpose of computerizing the analysis Livesley [12] has modified the form of the stability functions as follows :-

$$\left. \begin{aligned}
 \phi_1 &= \alpha \cot \alpha = \frac{2S(1+C) - \pi^2 \rho}{2S(1+C)} \\
 \phi_2 &= \frac{\alpha^2}{3(1-\phi_1)} = \frac{S(1+C)}{6} \\
 \phi_3 &= \frac{3\phi_2 + \phi_1}{4} = \frac{S}{4} \\
 \phi_4 &= \frac{3\phi_2 - \phi_1}{2} = \frac{SC}{2} \\
 \phi_5 &= \phi_1 \phi_2 = \frac{2S(1+C) - \pi^2 \rho}{L}
 \end{aligned} \right\} \quad (2.14)$$

Using these functions, and including the effect of axial deformation, the stiffness matrix for a member can be written as :-

$$K = \begin{bmatrix} \frac{EA}{L} & & & & & & \\ & 0 & \frac{12EI\phi_5}{L^3} & & & & \\ & 0 & \frac{6EI\phi_2}{L^2} & \frac{4EI\phi_3}{L} & & & \\ -\frac{EA}{L} & & 0 & 0 & \frac{EA}{L} & & \\ & 0 & \frac{-12EI\phi_5}{L^3} & \frac{-6EI\phi_2}{L^2} & 0 & \frac{12EI\phi_5}{L^3} & \\ & 0 & \frac{6EI\phi_2}{L^2} & \frac{2EI\phi_4}{L} & 0 & \frac{-6EI\phi_2}{L^2} & \frac{4EI\phi_3}{L} \end{bmatrix} \quad (2.15)$$

The function $\phi_1 = \alpha \cot \alpha$ gives singular values at $\rho = 4, 16, 36, \text{etc.}$ For this reason Livesly derived a method whereby this function is calculated as the sum of power series in ρ and rational function. Thus ϕ_1 is calculated as:

$$\phi_1 = \frac{64 - 60\rho + 5\rho^2}{(16 - \rho)(4 - \rho)} - \sum_{n=1}^7 \frac{a_n \rho^n}{2^{2n}} \quad (2.16)$$

Where :-

$$a_1 = 1.57973627, \quad a_2 = 0.158588587, \quad a_3 = 0.02748899,$$

$$a_4 = 0.00547540, \quad a_5 = 0.00115281, \quad a_6 = 0.00024908.$$

$$a_7 = 0.00005452.$$

2.2.2 FINITE ELEMENT METHOD :-

The finite element method is essentially a method by which the whole structure is divided into a number of "finite elements". These elements are assumed to be interconnected at a finite number of nodal points or joints.

For the plane frame 2D-beam column element shown in Figure (2.3) is used, the nodal displacement vector U and the load vector R are given by

$$U = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \quad R = \begin{Bmatrix} P_1 \\ V_1 \\ M_1 \\ P_2 \\ V_2 \\ M_2 \end{Bmatrix} \quad (2.17)$$

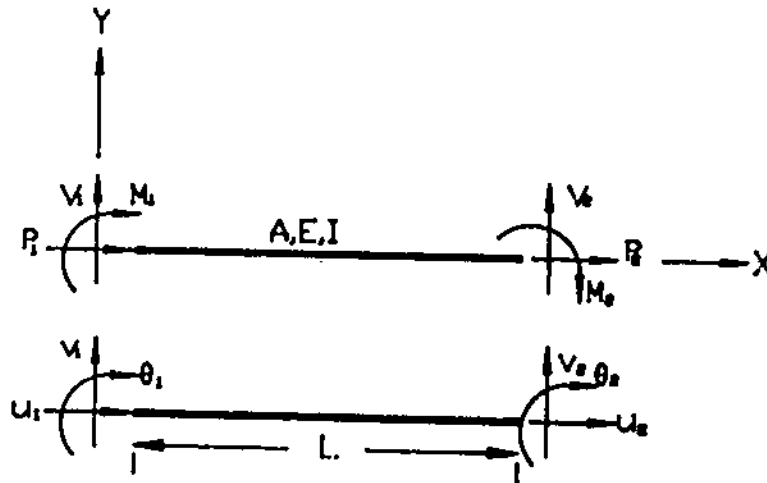


FIG.(2-3)

The strain energy U_i is given by

$$U_i = \frac{1}{2} \int_v E \epsilon_x^2 dv \quad (2.18)$$

Where ϵ_x is the axial strain in the element and, it is given by

$$\epsilon_x = \frac{\delta u}{\delta x} + \frac{1}{2} \left(\frac{\delta v}{\delta x} \right)^2 - y \frac{\delta^2 v}{\delta x^2} \quad (2.19)$$

Substituting eq. (2.19) into 2.18 yields

$$U_i = \frac{E}{2} \int_0^l \left(\left(\frac{\delta u}{\delta x} \right)^2 + \frac{\delta^2 v}{\delta x^2} y^2 + \frac{1}{4} \left(\frac{\delta v}{\delta x} \right)^4 - \frac{2\delta u}{\delta x} \frac{\delta^2 v}{\delta x^2} y - \frac{\delta^2 v}{\delta x^2} \left(\frac{\delta v}{\delta x} \right)^2 y + \frac{\delta u}{\delta x} \left(\frac{\delta v}{\delta x} \right)^2 \right) dx dA \quad (2.20)$$

Neglecting higher-order terms, and using $\int y dA = 0$ yield

$$U_i = \frac{EA}{2} \int_0^l \left(\frac{\delta u}{\delta x} \right)^2 dx + \frac{EI}{2} \int_0^l \left(\frac{\delta^2 v}{\delta x^2} \right)^2 dx + \frac{EA}{2} \int_0^l \frac{\delta u}{\delta x} \left(\frac{\delta v}{\delta x} \right)^2 dx \quad (2.21)$$

To proceed, U_i should be given as a function of nodal displacements, so the following approximate displacement functions are assumed:

$$u = \left(1 - \frac{x}{l} \right) u_1 + \left(\frac{x}{l} \right) u_2$$

$$v = \left(1 - \frac{3x^2}{l^2} + \frac{3x^3}{l^3} \right) v_1 + \left(\frac{3x^2}{l^2} - \frac{3x^3}{l^3} \right) v_2 + \left(x + \frac{x^3}{l^2} - \frac{2x^2}{l} \right) \theta_1 + \left(\frac{x^2}{l^2} - \frac{x}{l} \right) \theta_2 \quad (2.22)$$

Using these functions and integrating eq.(2.21) gives U_i as a function of nodal displacements. According to the Castigliano's first theorem

$$R_i = \frac{\delta U_i}{\delta u_i} \quad (2.23)$$

The stiffness matrix of the element is determined, which is found to be a summation of two matrices

$$K = K_e + K_g \quad (2.24)$$

Where :

K_e : The conventional linear elastic stiffness matrix

K_g : The geometric stiffness matrix, also called the initial stress stiffness matrix, and it is given by

$$K_G = \frac{P}{l} \begin{bmatrix} 0 & & & & & \\ 0 & \frac{6}{5} \frac{L}{10} & & & & \\ 0 & \frac{L}{10} & \frac{2L^2}{15} & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -\frac{6}{5} \frac{L}{10} & -\frac{L}{10} & 0 & \frac{6}{5} & \\ 0 & \frac{L}{10} & -\frac{L^2}{30} & 0 & -\frac{L}{10} & \frac{2L^2}{15} \end{bmatrix} \quad (2.25)$$

2.3 IMPLEMENTATION :

In the previous sections, the stiffness matrix for individual members subjected to axial load were derived, using stability functions in one approach, and the finite element method in the other. The stability functions approach is described as an exact solution under the assumptions that the shear deformation effect is ignored and the member is prismatic, while the finite element approach is approximate due to the following :-

1. The strain-displacement relationship in equation 2.19 neglects higher order terms.
2. The displacement functions used to relate strain energy in the member to its nodal displacement are the hermition polynomials eq. (2.22)

The finite element approach often favored because it avoids the use of the non-linear stability functions which are numerically sensitive near the limit of stability and particularly difficult to handle at that stage if expressed in the original form (2.11 and 2.12')

Once the individual member stiffness matrix is computed, it is then added to the global stiffness matrix using direct stiffness addition.

2.3.1 FORMULATION OF THE GLOBAL STIFFNESS MATRIX :

An iterative procedure is employed for the determination of the global stiffness matrix for a specific external load vector, since the axial forces in the members are unknown to begin with. The steps required for a complete non-linear analysis of a frame for a given load factor λ are:-

1. Assume the axial forces in the members to be zero.
2. Construct the global stiffness K using direct stiffness method.
3. Solve the joint equilibrium equations $KU=R$ for the displacements U , and then get member forces.
4. Calculate the difference between two successive sets of axial forces, or displacements, and when the difference is within a specific tolerance the process is stopped.
5. Use the new axial forces in the members to compute the member stiffness.
6. Repeat the process starting from step 2.

The global stiffness matrix that results at the end

of the above process is the tangent stiffness of the structure at a particular level of loading λ . At the critical loading level λ_c the following conditions is reached:

$$K(\lambda) U = 0$$

This equation represents what is called an eigen value problem. The solution of it is represented by N roots which are the eigen values of the equation, where N is the order of the stiffness matrix, of these the smallest eigen value is the only one required and it represents the critical load factor λ_c . To predict λ_c quickly and effectively, a general effective eigen value algorithm may be used.

2.3.2 A SOLUTION METHOD FOR EIGENVALUE PROBLEM:

Although many solution techniques for eigen problems have been proposed, it must be realized that there does not as yet exist a single algorithm that is always most effective [45]. In the case of elastic stability analysis where only the lowest eigen value is required to estimate λ_c , the determinant search iteration method seems to be the most effective technique.

2.3.2.1 SOLUTION ALGORITHM FOR DETERMINANT SEARCH
METHOD :

Eq. (2.23) can be rewritten as

$$(K(\lambda))U = 0$$

or

$$(K - \lambda K_G) = 0$$

The algorithm aims to get the smallest eigen value λ_c . If λ_k and λ_{k-1} are two approximations for λ_c and $\lambda_{k-1} < \lambda_k \leq \lambda_c$, as shown in Fig. (2.4).

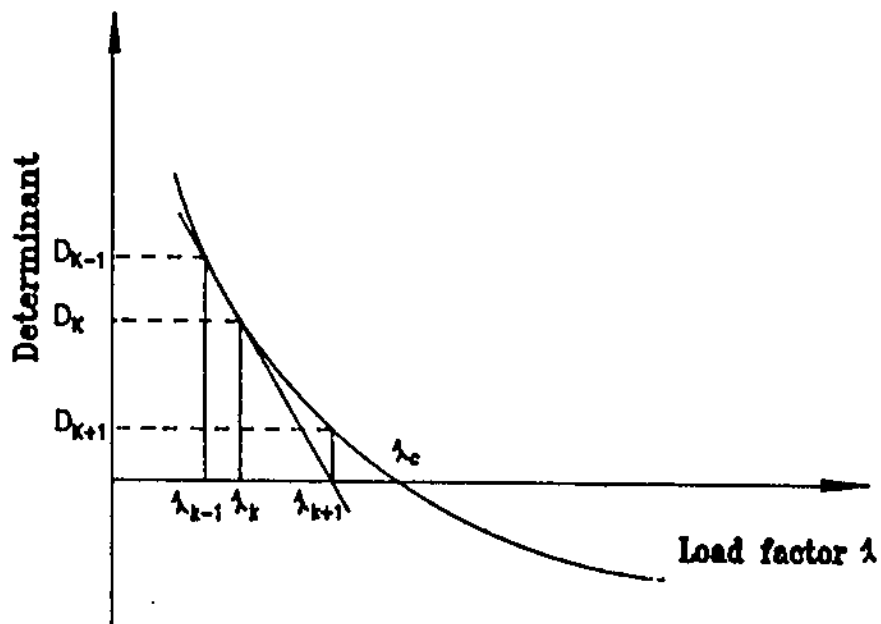


Fig.(2.4)

It is then possible to obtain a quick convergence to λ_c by using secant iteration in which the next estimate λ_{k+1} is calculated by

$$\lambda_{k+1} = \lambda_k - \frac{D_k}{D_k - D_{k-1}} (\lambda_k - \lambda_{k-1}) \quad (2.26)$$

Where D_k is The determinant of K corresponding to λ_k .

Carrying out a non-linear analysis for the predicted load factor λ_{k+1} , the global stiffness matrix K under this load factor can be formulated, and then reduced to lower and upper forms LS .

Application of the sturm sequence technique makes it possible to compute the determinant, and the 'signcount' ($nsign$) of K .

The determinant D_{k+1} is found as the product of the diagonal elements of S , while ($nsign$) is the number of the negative diagonal elements of S . $Nsign$ determines the number of critical load factors (i.e eigen values) lying between $\lambda=0$ and trial value λ_{k+1} . In all the iterations λ_k is kept less than λ_c , in other words D_k is always greater than zero, and ($nsign$) $_k$ is always equal to zero, as ($nsign$) $_{k+1}$ becomes bigger than zero the bisection method is used to force convergence to the smallest eigen value λ_c .

2.3.3 GENERAL ALGORITHM FOR ELASTIC STABILITY ANALYSIS:

A general algorithm for elastic stability analysis can be easily developed utilizing the iterative non-linear elastic analysis algorithm described in section (2.3.1), together with the iterative eigen solution algorithm described in section (2.3.2.1).

The steps required for such algorithm are shown in Sec. (3.1.2).

2.4 ELASTIC-PLASTIC ANALYSIS (LIMIT ANALYSIS):

It was assumed in the elastic stability analysis that the material remains elastic irrespective of the stress level. In practical situations the material remains elastic up to a certain point, where plasticity sets in.

For elastic-plastic analysis, material is assumed to deform in the idealized manner shown in Fig. (2.5). In this figure the strain and the stress are proportional up to point A at which the strain increases indefinitely without any further increase in the stress.

The resisting moment of the section when the stress reaches point A over the entire section tension and

compression is called the plastic moment capacity of the section (MP).

When the applied moment at any end of the member reaches MP, a plastic hinge is located at that end indicating that the member at that end can not resist any extra moment and it is free to rotate.

In this case the stiffness matrix of the member should be modified to account for this hinge.

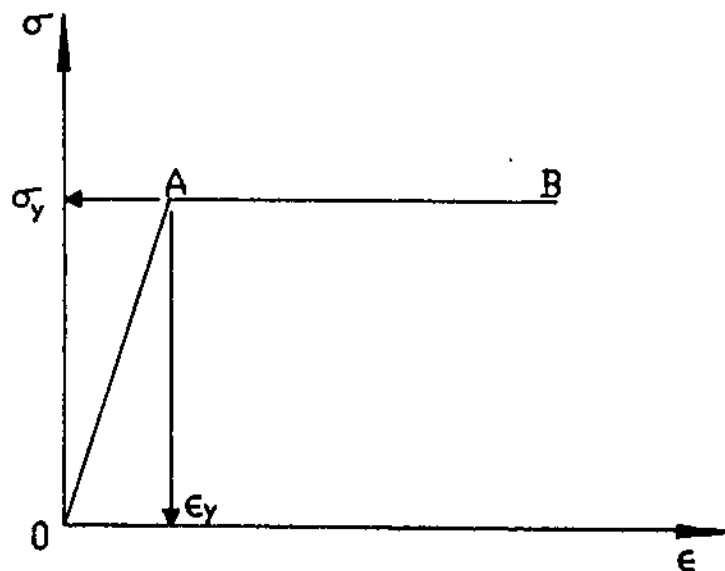


Fig.(2.5) Idealized stress-strain curve.

The effect of axial force on member stiffness discussed previously can also be included in the elastic-plastic analysis. So the failure load factor due to combined effect of plasticity and stability can be determined.

If the hinge is at end j then

$$K_e = \begin{bmatrix} \frac{EA}{l} & & & & & \\ 0 & \frac{3EI}{l^3} & & & & \\ 0 & \frac{3EI}{l^3} & \frac{3EI}{l} & & & \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & & \\ 0 & -\frac{3EI}{l^3} & -\frac{3EI}{l} & 0 & \frac{3EI}{l^3} & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.29)$$

and

$$K_G = \frac{P}{l} \begin{bmatrix} 0 & & & & & \\ 0 & \frac{6}{5} & & & & \\ 0 & \frac{6}{5} & \frac{l^2}{5} & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -\frac{6}{5} & -\frac{l}{5} & 0 & \frac{6}{5} & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.30)$$

While if there are two hinges in the two ends K_e and K_G is given as for a truss member .

2.4.2 EFFECT OF AXIAL FORCE ON THE PLASTIC MOMENT CAPACITY:

It is known that in the presence of high axial compressive or tensile forces the plastic moment of the section (MP) decreases.

For wide flange I-sections used in steel construction, the approximate interaction curve Fig. (2.6) is given for bending about major axis. This curve can be described by the following equations:-

$$\text{For } P/P_y \leq 0.15 \quad M_{Pc} = MP$$

$$\text{For } P/P_y > 0.15 \quad M_{Pc}/MP = 1.18 (1 - P/P_y)$$

Where :

M_{Pc} : Reduce plastic moment .

MP : Plastic moment when $P=0$

P : axial force in the member

P_y : Ultimate force $\sigma_y \times \text{Area}$.

2.4.3 CALCULATION OF FAILURE LOADS:

To computerize the elastic-plastic analysis, step by step procedure is used in which a series of elastic analyses using the direct stiffness method are performed. The procedure is carried out by adding increments of load to induce one hinge after the other. Once a plastic hinge is detected and inserted in the frame, the load factor at which the next hinge would form is extrapolated and applied to the frame. The frame is then analyzed utilizing the modified stiffness matrix in section (2.4.1) for members with plastic

hinges. the procedure is continued up to failure. this is indicated by :

1. Singularity of the stiffness matrix.
2. very large deflections indicating the formation of a mechanism.

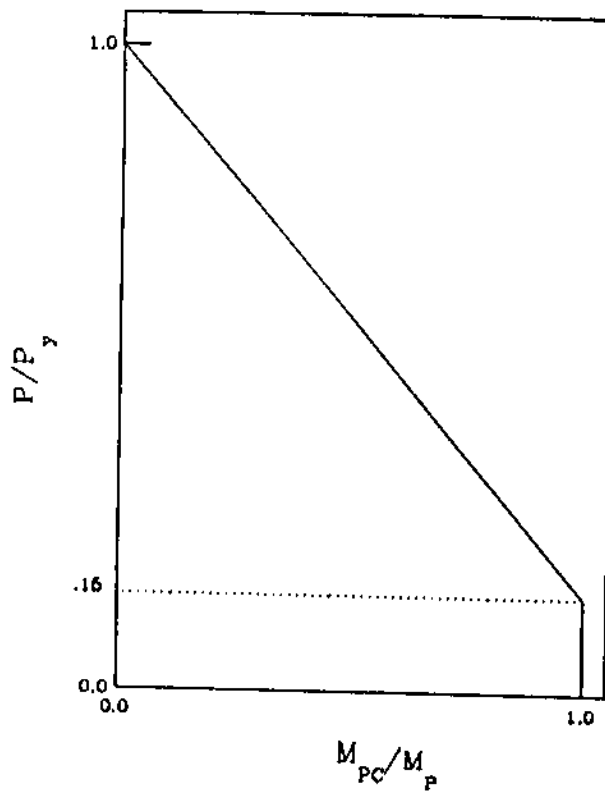


Fig.(2.6):Interaction curve for steel I-sections
(bending about major axis)

CHAPTER THREE
COMPUTER PROGRAMS

CHAPTER III
COMPUTER PROGRAMS

Two basic computer programs are developed. They are ESAP, elastic stability, analysis program and, PLAS, elastic-plastic analysis program. The programs are written using FORTRAN-77 for the VAX-8700 located in the Faculty of Engineering and Technology at the University of Jordan.

ESAP is used to find the critical load factor (λ_c) for plan frames, and, the corresponding effective lengths for compression members, while PLAS is used to find the plastic load factor (λ_p) with the member forces at this load factor.

A detailed description for the two programs separately, are presented in this chapter.

3.1 ELASTIC STABILITY ANALYSIS PROGRAM FOR PLANE FRAMES:

As it was shown, the aim of this program is to find the critical load factor λ_c for frame acting as single unit. Based on the axial force in each member at the instability

stage estimates are made for the effective lengths of the compression members in this frame.

The computation procedure described in section (2.3) is implemented here to develop this program. Two versions of the program were developed using the stability functions approach in the computation procedure for first version, and the alternative finite element approach for the second.

3.1.1 PROGRAM CAPACITY :

ESAP was written in FORTRAN-77 with dynamic storage allocation for major arrays in blank COMMON.

This removes the limitation on the number of joints, members, etc. and simply imposes a limitation on the total in-core storage. This overall limitation is set at compilation time depending on the disk quota or addressable storage by the fortran compiler on PC'S. The global stiffness matrix usually occupies the largest part of storage inspite of its storage in skyline form.

The total amount of storage is as follows :

$$N_{\text{Total}} = 5N_{\text{Join}} + 5N_{\text{ELEM}} + 6N_{\text{DF}} + 4N_{\text{LODN}} + 5N_{\text{LODEL}} + 3N_{\text{MATS}} + 2N_{\text{WK}}$$

Where :

NJoin : number of joints

NELEM : number of element

NMATS : number of materials and cross-sectional properties groups for the elements.

NDF : number of degrees of freedom (conservatively $N_{Join} \times 3$)

NLODN : number of loaded joints

NLODEL : number of loaded elements

NWK : number of elements that needed to be stored from the over all stiffness matrix in the skyline form.

This N_{total} is compared by the program with the set capacity and execution is halted when N_{total} exceeds this capacity.

3.1.2 INTERNAL ORGANIZATION OF ESAP :

ESAP is divided into four major subroutines as shown in Fig. (3.1) they are:

- (1) **MAIN1** : This is the first routine to be called by ESAP. Its main job is to read, generate and print out data required to

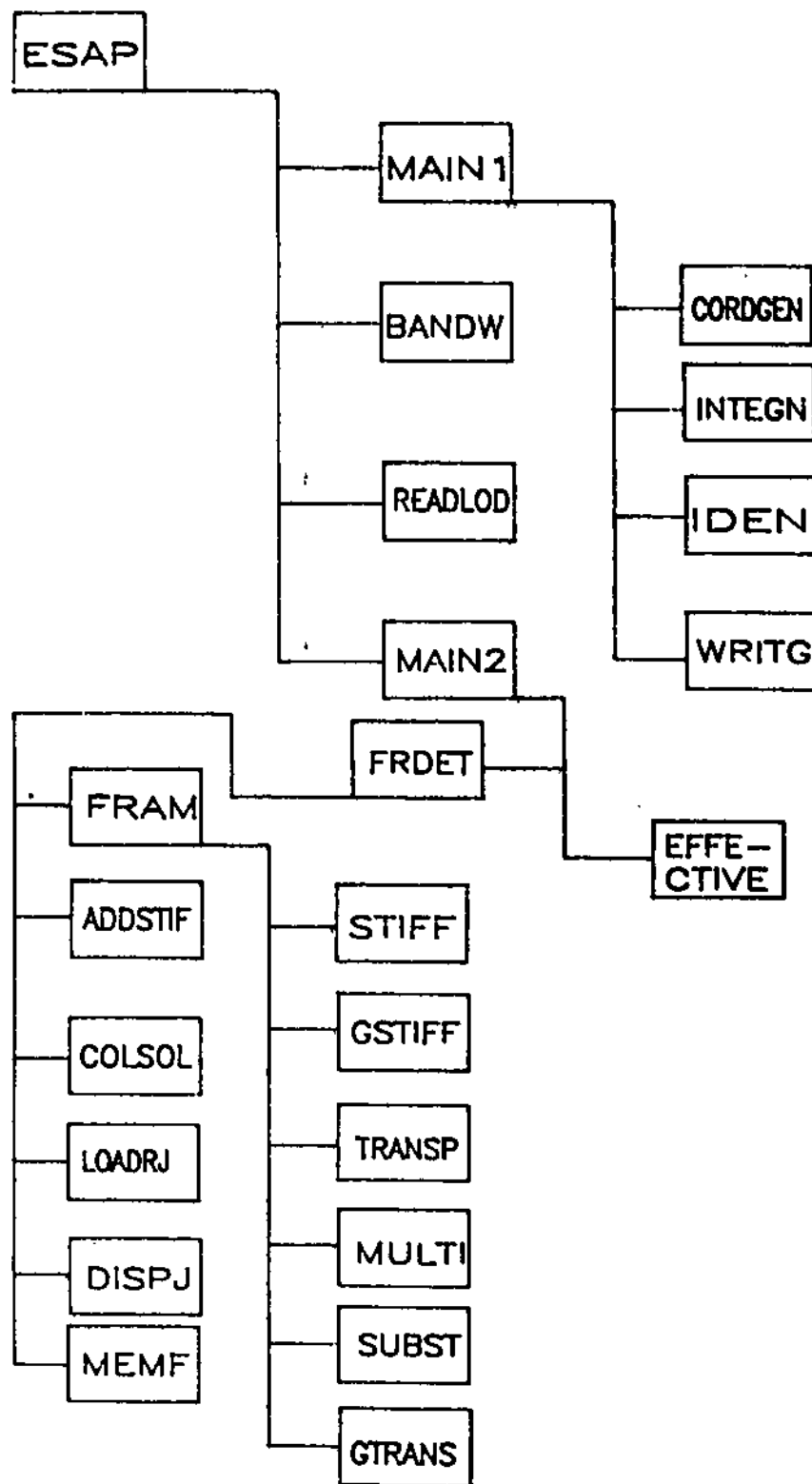
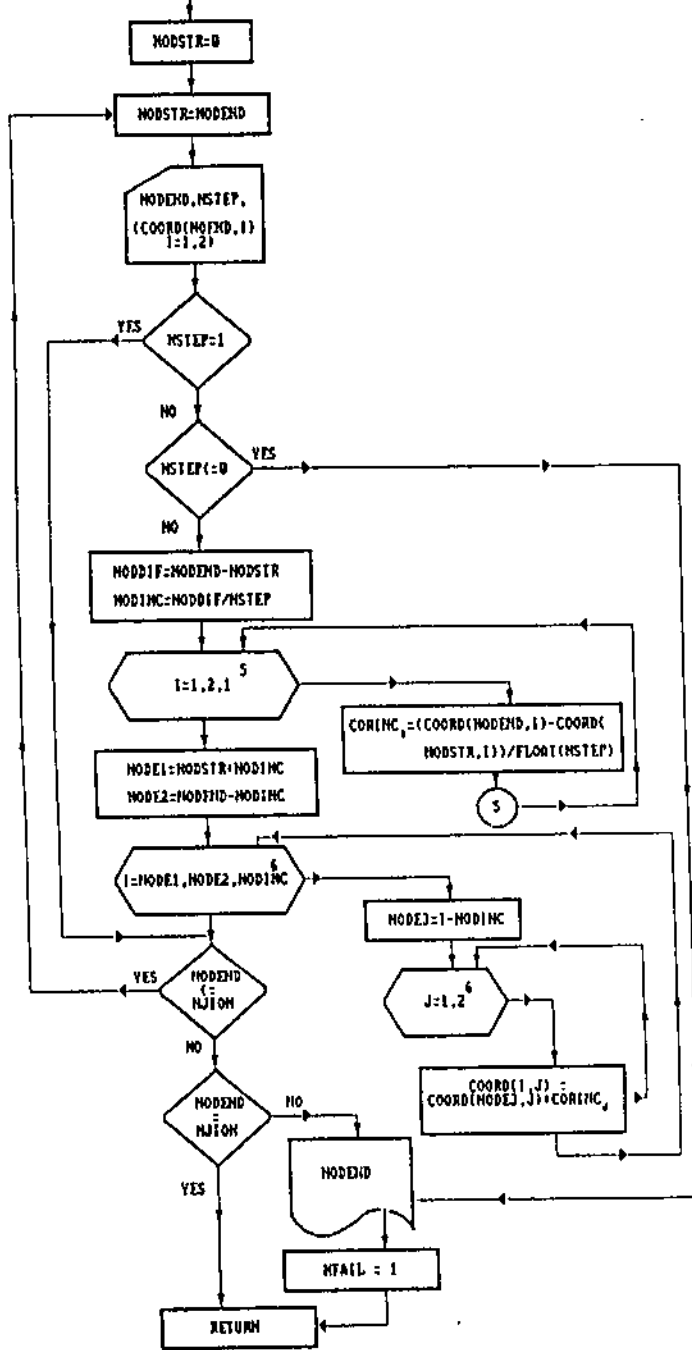


Fig.(3-1) Internal Organization of ' ESAP ' .

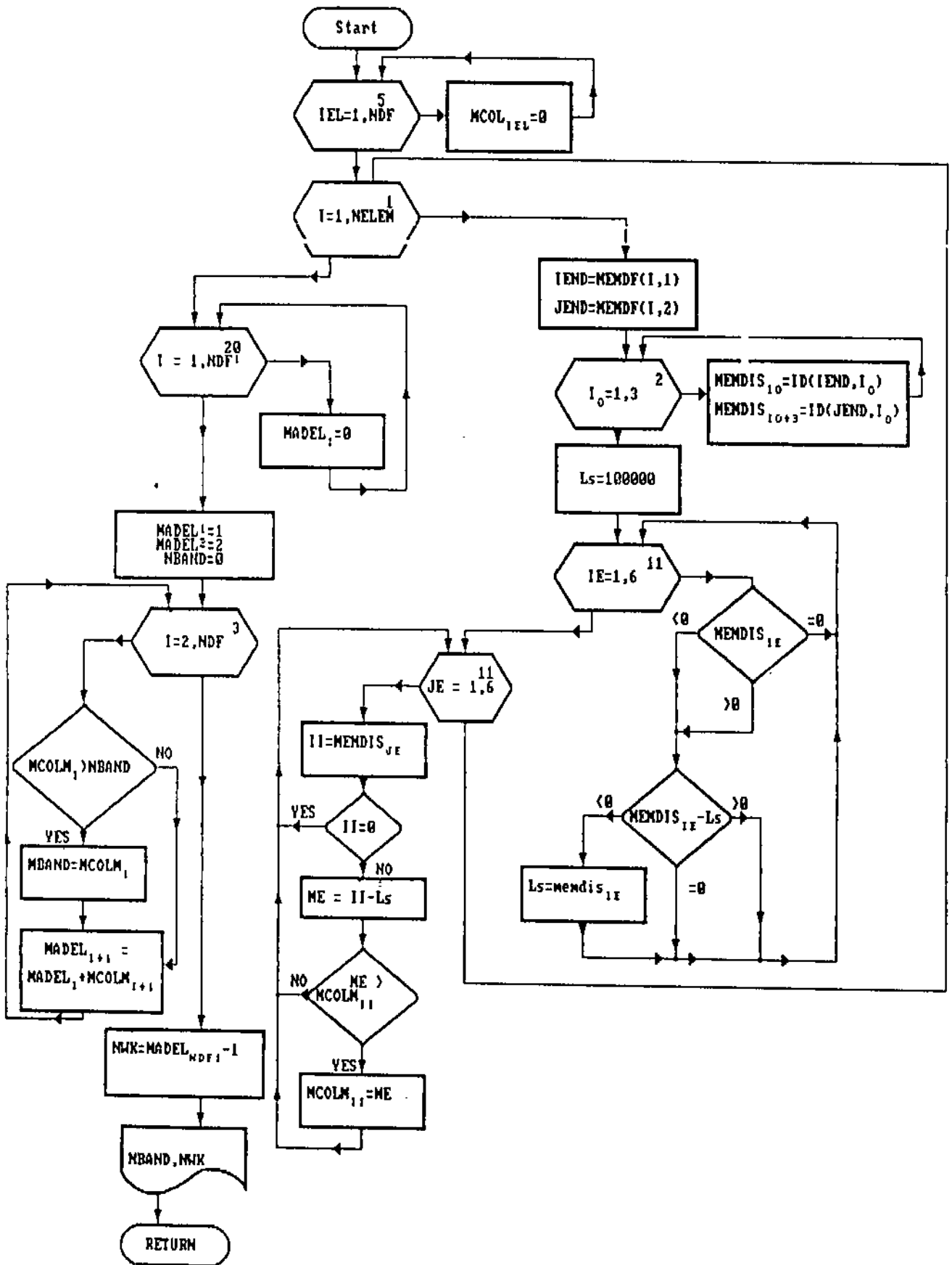
describe the problem, to do so MAIN1 calls four other subroutines respectively, they are :

- a) Subroutine COORDGEN : This subroutine is used to read and generate data about joints coordinate. Generation of joint coordinate is restricted to regular sets of joints equally spaced at the same line. Fig (3.2) shows a detailed flow chart for this routine.
- b) Subroutine INTGEN : This subroutine is used to read and generate data for all regular sets of data about members, incidence and members' material and cross-section properties. INTGEN is similar in all aspects to COORDGEN, and the flow chart shown in Fig. (3.2) can be easily adjusted to be used for INTGEN.
- c) Subroutine IDEN : This is a simple routine used to create the identification matrix for the structure which tabulates the degrees of freedom for all joints.



Fig(3.2): FLOW CHART FOR COORDZGH

- d) Subroutine WRITG : It is a subroutine which prints data read in the previous routine to the output file.
- (2) Subroutine BANDW : This routine is used to find the maximum half band width of the stiffness matrix. The number of non zero elements above each diagonal is stored in a vector whose length is equal to NDF. BANDW creates another array in which the addresses of the diagonal elements of the full stiffness matrix are recorded when it is stored in a one dimensional array. This array is essential in the assemblage of the stiffness matrix and its reduction. A detailed flow chart for BANDW is shown in Fig.(3.3).
- (3) Subroutine READLOD : This routine is used simply to read and store data on loaded joints and members. This data is then used in Subroutine (LOADRJS) to calculate the load vector.
- (4) Subroutine MAIN2: This subroutine constitutes the main body of the program. Elastic stability analysis is carried out in this routine, and the other routine called by it (FRDET).



Fig(3.3): Flowchart for BANDW routine

A general algorithm for MAIN2 is given in sec. (2.3).

The following steps are performed in MAIN2 :

- a) Choose two load factors λ_k , λ_{k-1} to start with.
- b) Use FRDET to carry out non linear analyses for these load factors. After these non-linear analyses are carried out the determinants of the stiffness matrix at load factors λ_k and λ_{k-1} are calculated together with an other quantity called (nsgn) which is the number of negative diagonal elements in the factorized matrix (LS) defining how many roots are left behind at a specific load factor, nsgn should always be zero to be sure that both λ_k or λ_{k-1} are less than λ_c .
- c) Use secant formula to estimate λ_{k+1} , then replace λ_k with λ_{k+1} and λ_{k-1} with λ_k .
- d) Use FRDET to carry out non-linear analysis for λ_k . If the structure is still stable then the analysis is possible and a convergence can be achieved in FRDET where D_k and nsgn are calculated. If convergence is not possible after a specific number of

iterations then the stiffness matrix at λ_k is ill condition. So λ_c is predicted and the program is stopped.

- e) Make sure that $\lambda_k < \lambda_c$ by checking that $(n\text{sign})_k = 0$, if not then bisection method is used to shift λ_k to a position where its less than λ_c , this is checked by repeating the process from step (d).
- f) Check two successive values of λ , if the difference is within a specific tolerance then the process is repeated starting from step c.
- j) Once the process is stopped, Subroutine EFFECTIVE is called to find the effective length factors for compression members.

SUBROUTINE FRDET :

As it is shown this routine is used to carry out non linear analysis for a specific load factor, and then to calculate the determinant and $(n\text{sign})$ of the stiffness matrix at this load factor utilizing the algorithm given in sec.(2.3). To do so FRDET calls the following subroutines :

- (1) Subroutine FRAM : This routine is used to calculate the stiffness matrix for each

member.

- (2) Subroutine ADDSTIFF : As each member stiffness is calculated, ADDSTIFF is called to add it to the structural stiffness matrix which stores it in the a compact form in which only non zero elements above diagonal, and the diagonal elements are stored in one dimensional array.
- (3) Subroutine FMEM : This routine is used to calculate member forces.
- (4) Subroutine COLSOL : This routine is used to solve the equilibrium equations in core using compact storage and a column reduction scheme. The stiffness matrix is factorize to LS form, and the determinant of the stiffness matrix is calculated as the product of the diagonal elements of the upper matrix S, also the number of negative diagonal elements (nsign) is computed.
- (5) Subroutine LOADRJS : This routine is used to form the load vector for a specific load factor λ .

3.2 ELASTIC-PLASTIC ANALYSIS PROGRAM OF PLANE FRAME (PLAS):

The purpose of this program is to find the plastic load factor λ_p utilizing the computational procedure described in section (2.4).

The effects of axial forces in the members are included by program, these are :

- (1) Effect of axial force in reducing plastic moment capacity of the section.
- (2) Effect of axial forces in the stiffness matrix of the members, i.e. geometric nonlinearity

3.2.1 INTERNAL ORGANIZATION OF PLAS:

As shown in Figure (3.4) PLAS is divided into four major subroutines , they are :

SUBROUTINES MAIN1, BANDW, READLOD:

These are the same ones used in ESAP except that READLOD, reads data about loaded joints and not about distributed member loads.

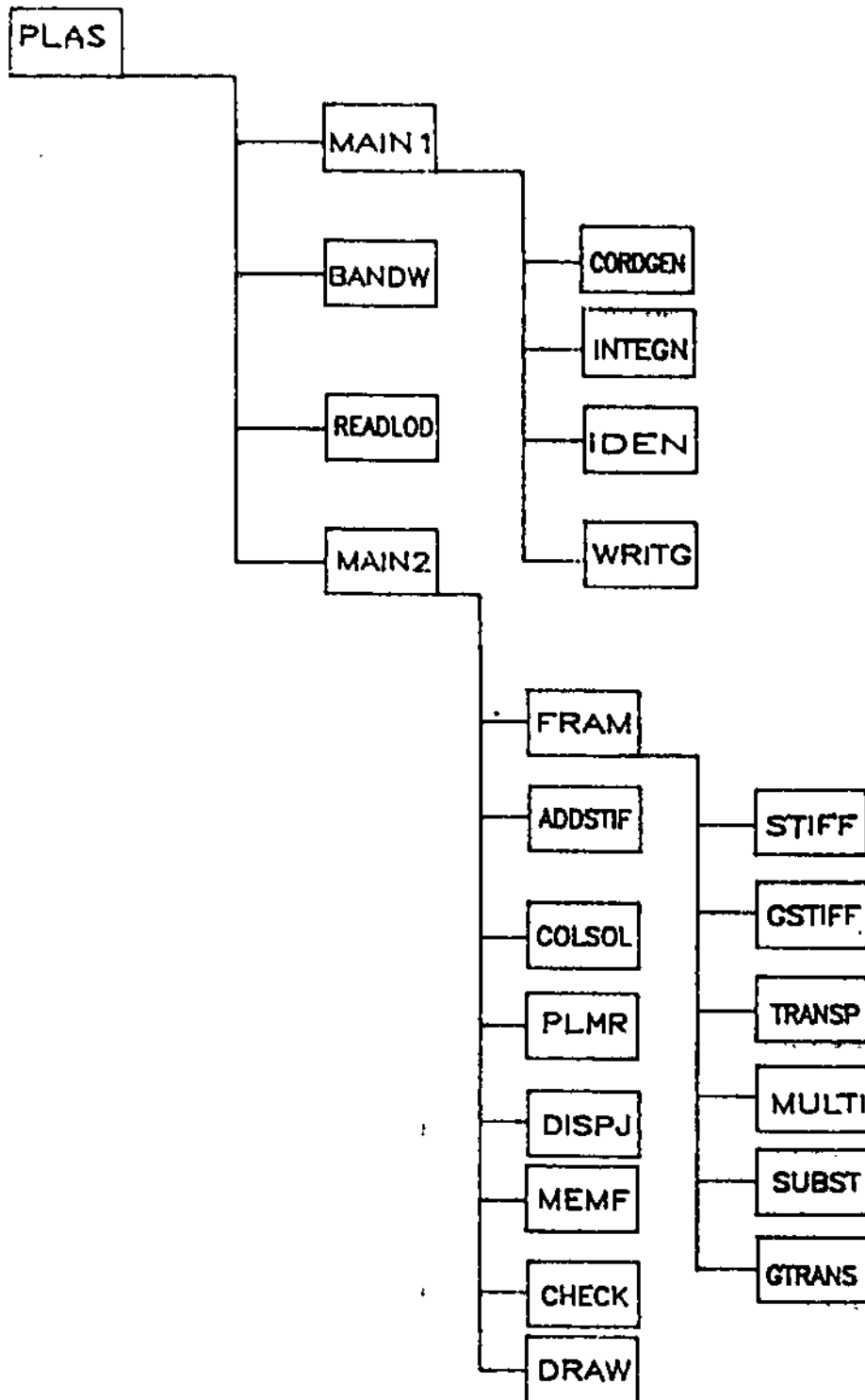


Fig. (3.4) Internal Organization of PLAS

SUBROUTINE MAIN2: -

In this routine step-by-step computational procedure for plastic analysis is performed to get the collapse load factor. In this procedure, analysis is performed each time to predict the load factor at which at least one plastic hinge forms, then the process is repeated until a mechanism is reached.

Steps required for elastic-plastic analysis by this procedure are:-

- (1) Select two load factors λ_1 and λ_2 , initially λ_1 is taken as zero, and λ_2 is taken as (1). This, of course, assumes that the structure remains in tact after the application of the working load.
- (2) Begin the external loop, a single loop will be completed only when at least one plastic hinge is formed in the structure.
- (3) Begin the internal loop. This loop is finished when it is found that the predicted load factor for the next hinge is true. The need of this loop arises from including the effect of axial forces in

reducing M_p , and the stiffness.

- (4) For λ_2 form K , solve equilibrium equations and get member forces.
- (5) Predict the load factor λ for the next plastic hinge to form. This step is done by looping over all elements in the structure and predicting for each element using the current load factor (and member forces), the factor at which the moment at a given end of the member reaches its plastic moment capacity. After this loop, the smallest load factor over these predicted is considered as the load factor at which the next hinge will form (λ).
- (6) Check if the predicted load factor is true, by calculating the difference between λ and λ_2 and comparing it with a present tolerance. If convergence is reached, proceed to step 7 else λ_2 is shifted to λ_1 , and λ is shifted to λ_2 . The plastic moments of the members are adjusted according to the calculated axial forces and the process then is repeated from step 3.
- (7) Determine how many hinges are formed at this cycle and their positions.

- (8) Calculate the cumulative load factor (λ_{com}) at this cycle as the summation of the true λ up to this cycle .
- (9) Check the predicted λ against λ_{com} and stop the process if it is found small enough.
- (10) The structure is adjusted to account for plastic hinges that have formed. A check of the stability of the structure is made by calculating the determinant of its stiffness matrix. When it is subjected to the initial loading vector (e.i., $\lambda = 1$), and if it is found stable the process is repeated from step 1.
- (11) When collapse is reached Subroutine DRAW is used to draw the structure and to locate the position of plastic hinges on it.

Through these steps MAIN2, calls a set of subroutines as shown in Fig. (3.4). These routines are similar to those called by ESAP and are discussed in Sec. (3.2) Except for subroutines PLMR and CHECK.

PLMR is the routine used in step 9 to reduce the plastic moment of the member due to the presence of axial forces as shown in Sec. 2.4.2 .

Subroutine CHECK is used in step 9 to determine the

possibility of hinge formation in a specific position. At a specific joint the number of plastic hinges that can form is restricted depending on the number of framing members. The number of possible hinges is thus equal to the number of framing member -1 except when the next hinge to form is the last one.

CHAPTER IV

EXAMPLES

In this chapter the efficiency and validity of the computer programs are checked. The accuracy of the finite element approach used for elastic stability analysis is also checked against stability function approach (exact solutions).

Analyses results for frames of different number of bays and storeys are presented. Each single frame is analyzed three times, two of them using the computer program ESAP based on the finite element and the stability functions approaches. In these two analyses λ_c for the frame is determined and then the effective length factor for each compression member in the frame is calculated. The third analysis is carried out for each frame using PLAS to get λ_p (plastic collapse load factor).

4.1 RESULTS OF THE ELASTIC STABILITY ANALYSIS:

Tables 4.1 - 4.8 shows the analyses results, which

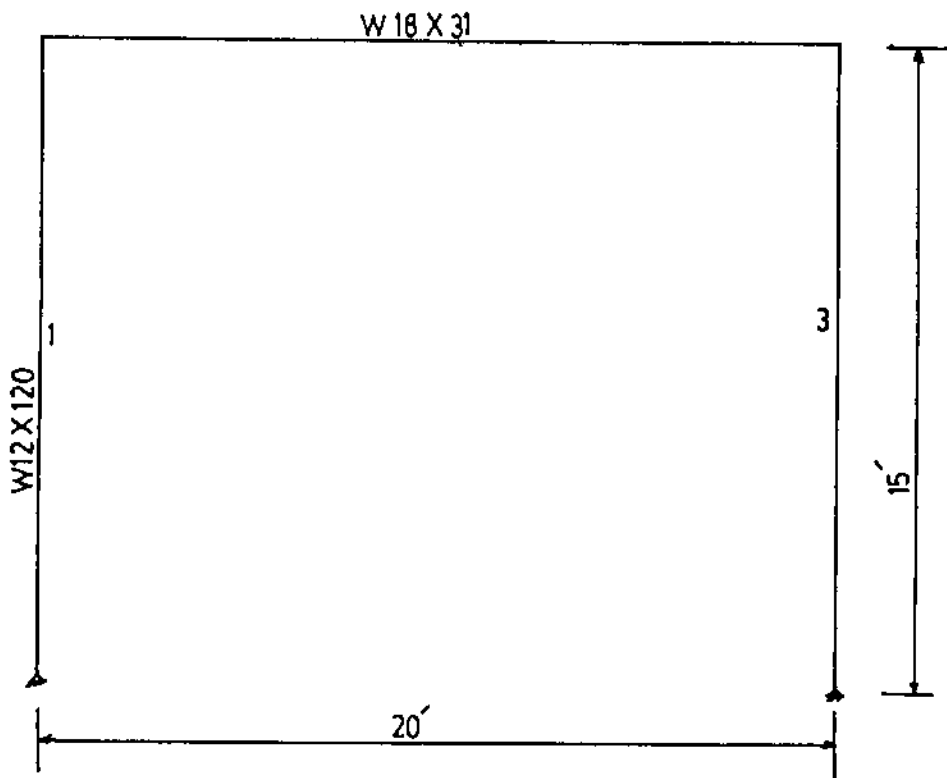


FIG.(4.1)FRAME 1

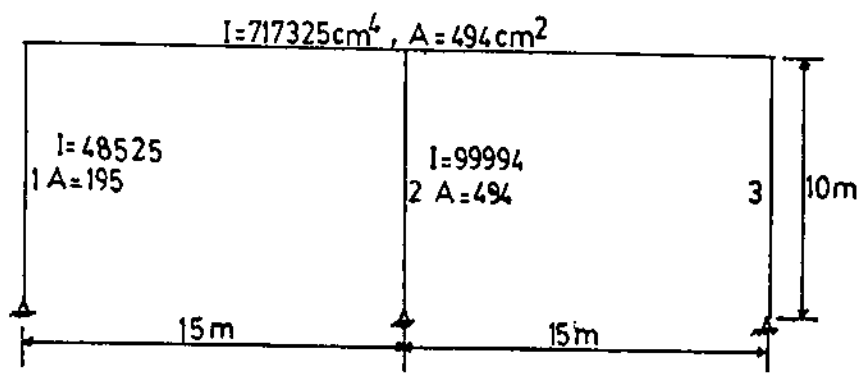


Fig.(4.5) Frame...5

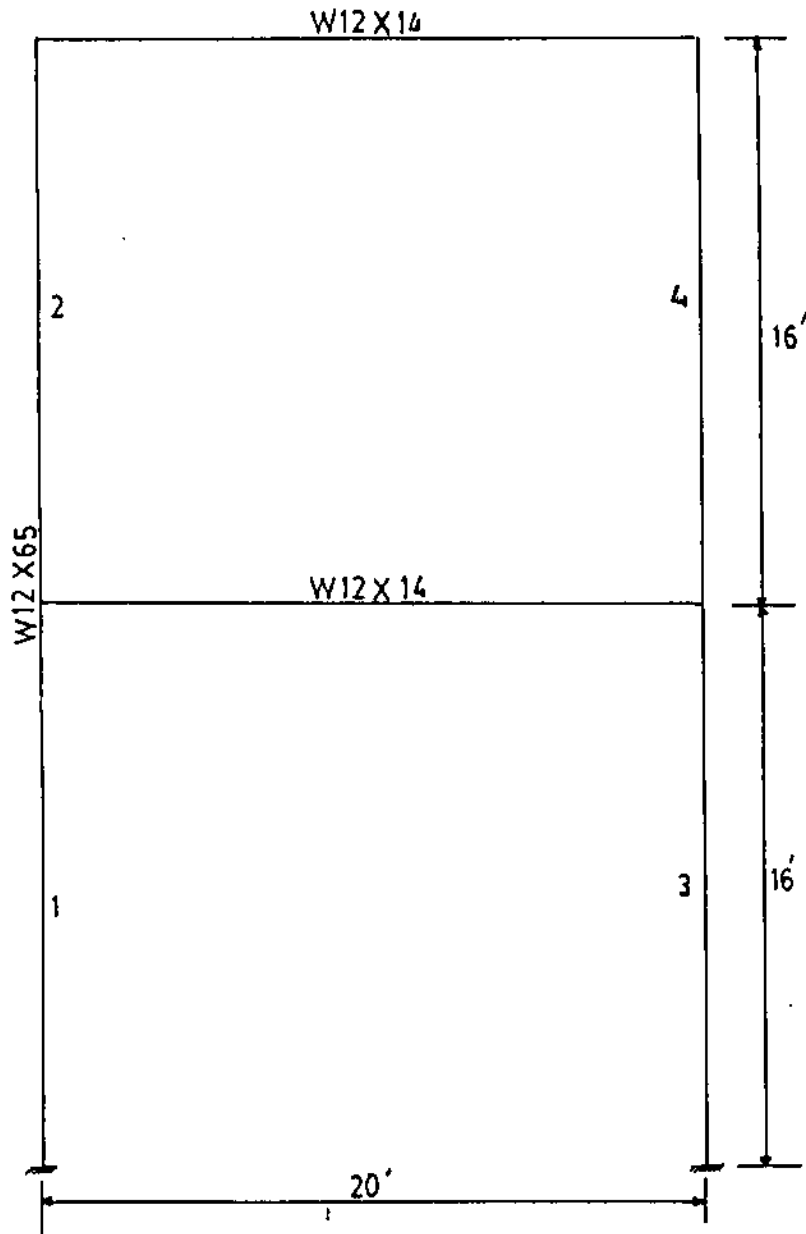


Fig.(4.2) Frame 2

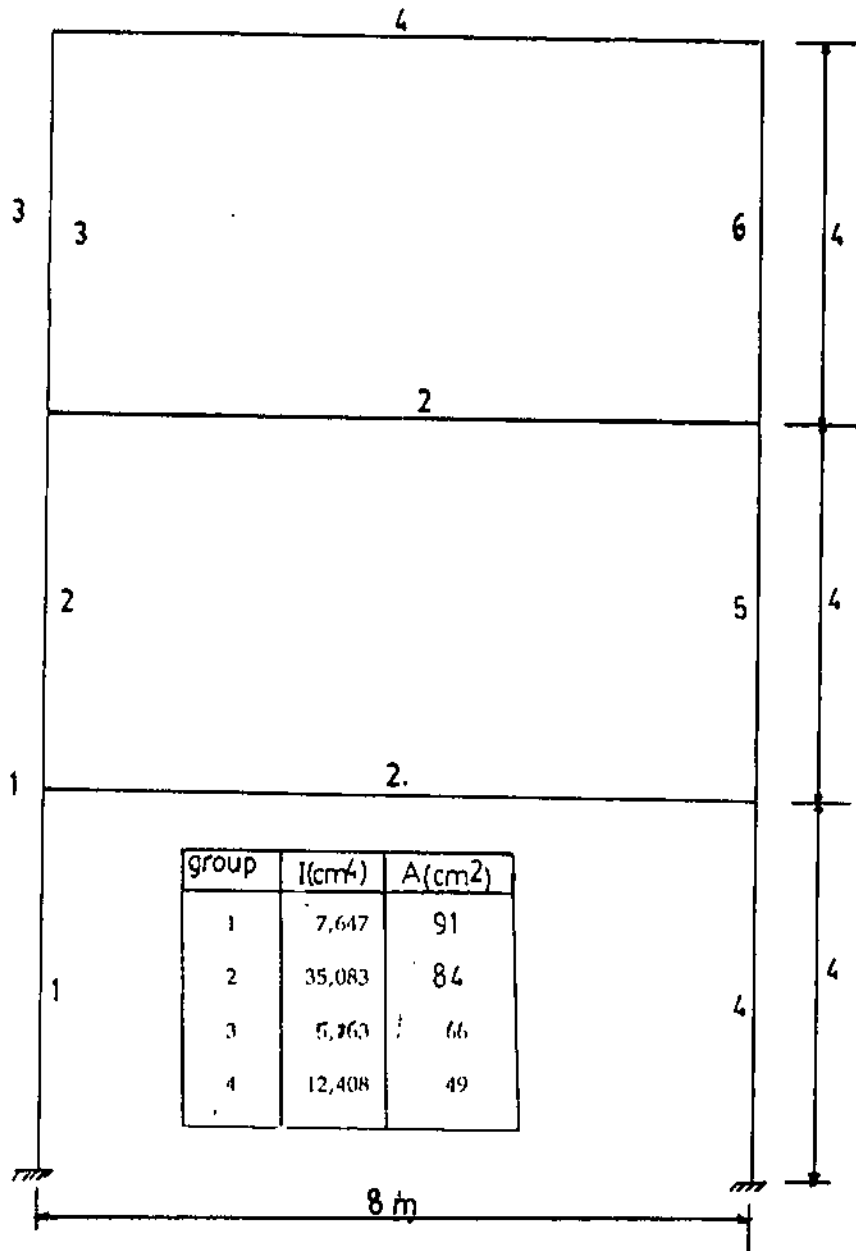


Fig.(4.3) Frame 3

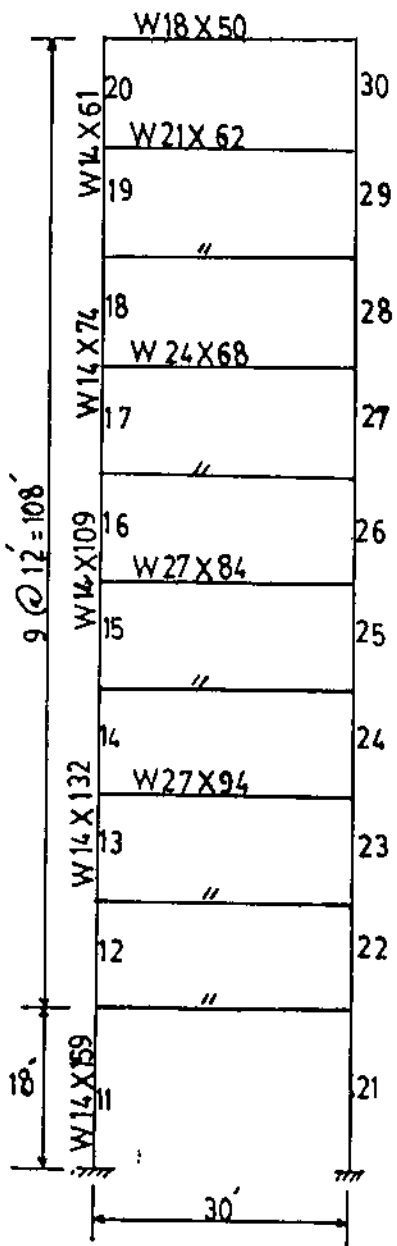


Fig.(4.4) Frame .. 4

group	I, in centimeters* (inches ⁴)	A, in square centimeters (square inches)
1	13,426 (323)	134 (20.8)
2	25,101 (603)	221 (34.3)
3	51,037 (1,226)	123 (19.1)
4	19,165 (460)	183 (28.4)
5	11,260 (271)	118 (18.3)
6	15,713 (378)	150 (23.2)
7	8,663 (208)	95 (14.7)
8	9,211 (221)	102 (15.8)
9	4,789 (115)	66 (10.2)
10	20,475 (492)	85 (13.2)

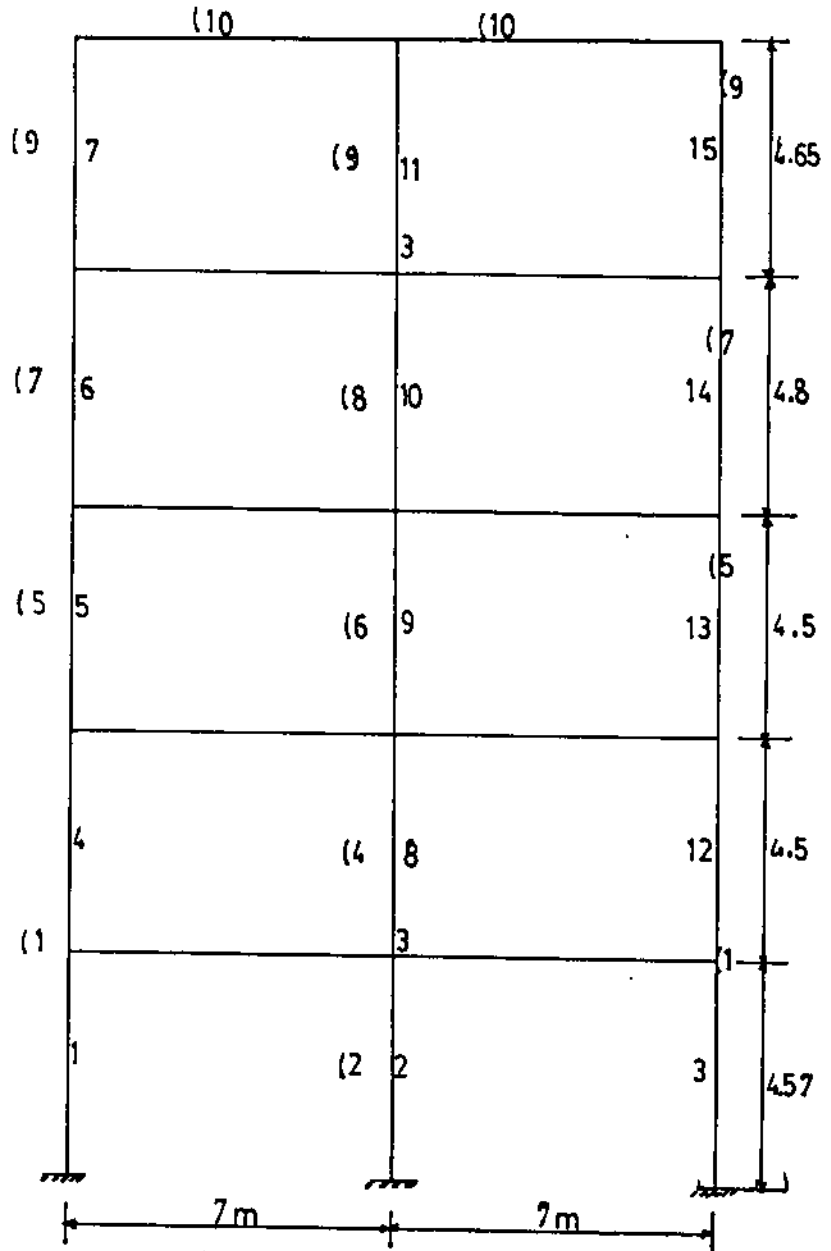


Fig.(4.6) Frame 6

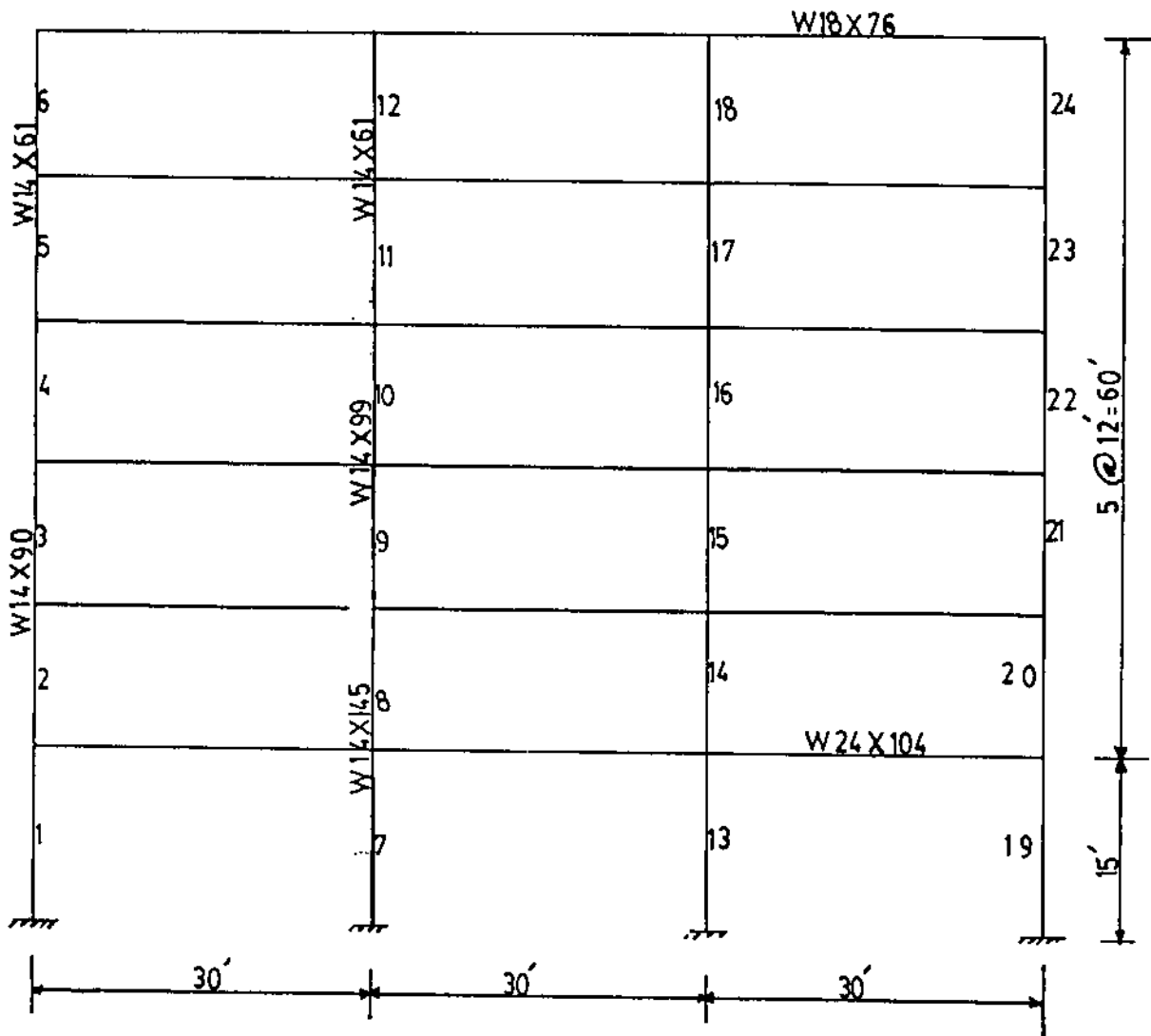


Fig.(4.7) Frame...7

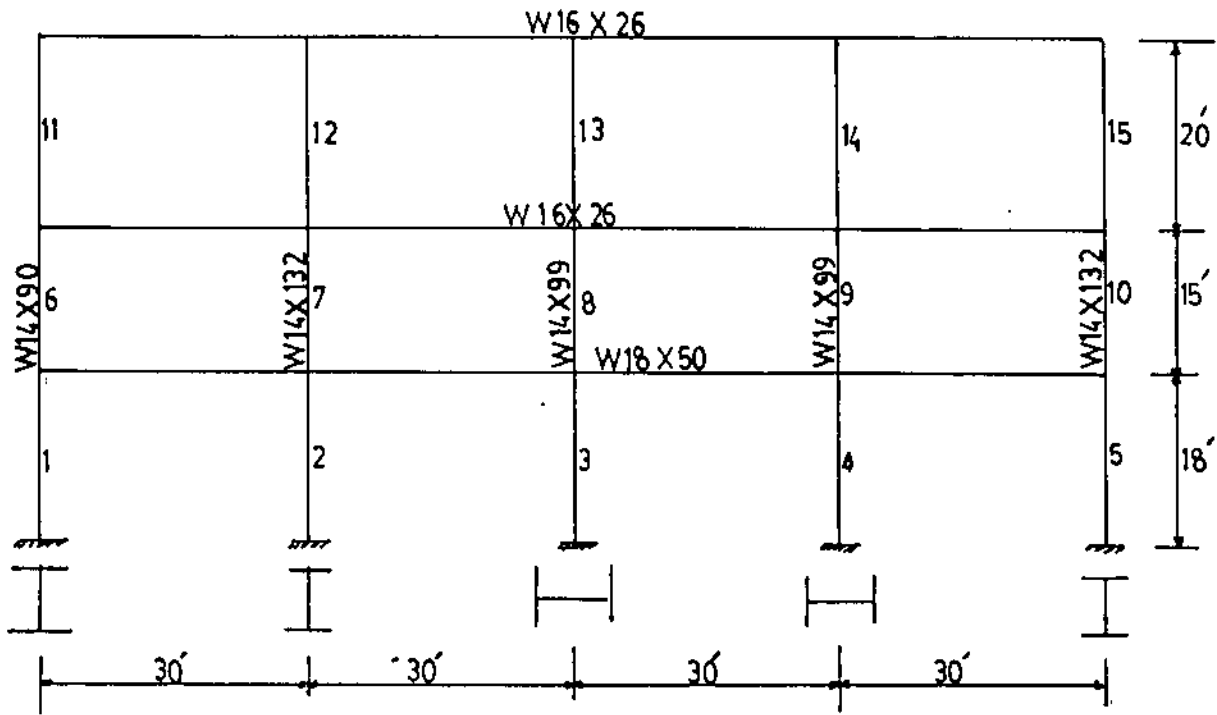


Fig.(4.8) Frame...8

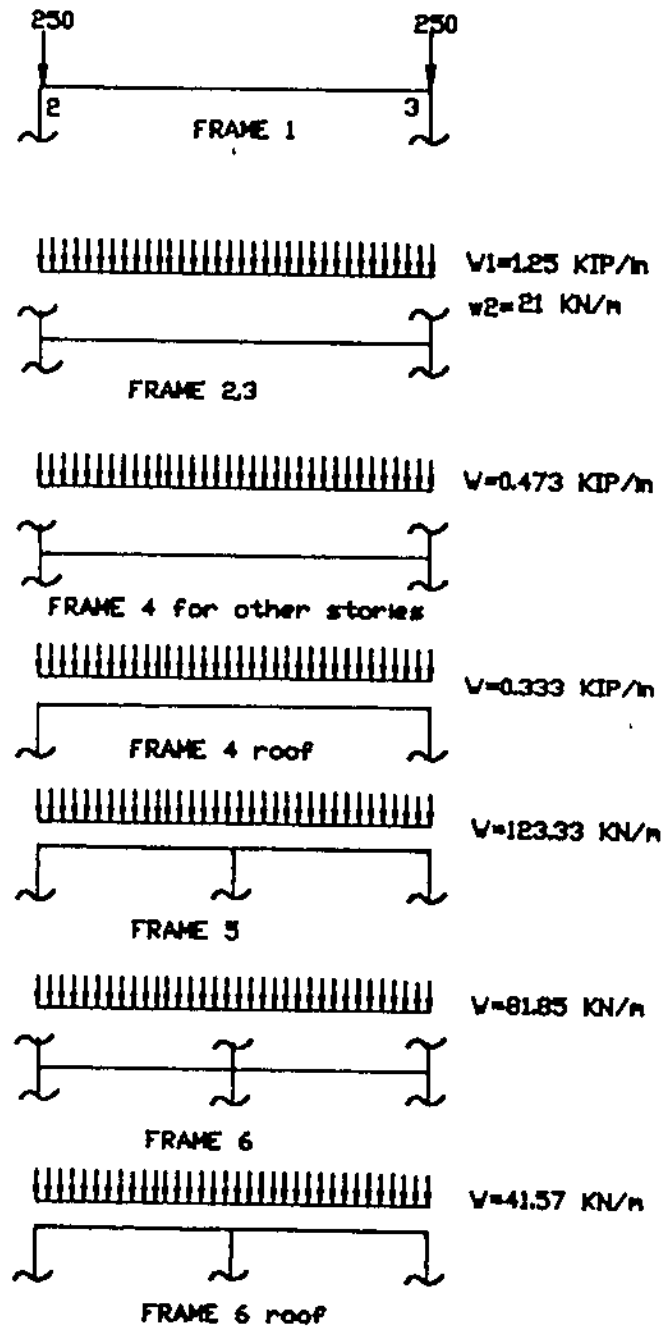
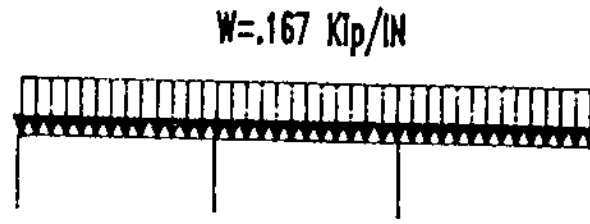
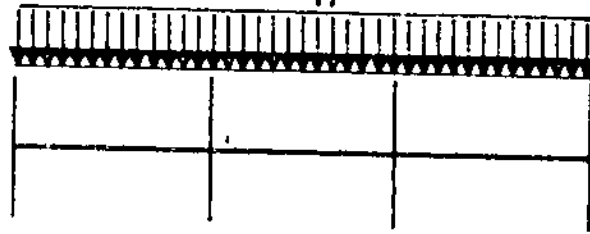


Fig.(4.9) Loading.

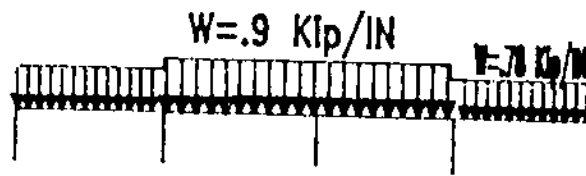


Frame 7 (Roof)

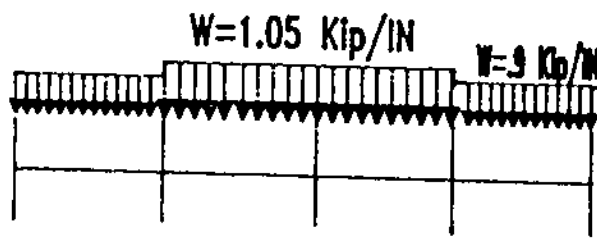
$W = 0.223 \text{ Kip/IN}$



Frame 7



Frame 8 (Roof)



Frame 8

Fig.(4.10)Loading

consist of λ_c and k-factor for each compression member in the frame, for the frames shown in figures 4.1-4.8, each frame is analyzed as a sway and non-sway frame, where full bracing is assumed, for the basic loading shown in figures (4.9,4.10).

Effective length factor for a member in the frame is calculated using the member's axial compression force found at λ_c for the whole frame. That is k is considered to be

$$K = \sqrt{\pi^2 EI / PL^2} \quad (4.1)$$

Where :

p = is the axial compression force in the member at λ_c .

4.1.1 Finite element Versus Stability functions:-

In tables 4.1 - 4.8 column 3 of the tables shows the effective length factor (K_f) as calculated using finite element approach, and column 4 gives the effective length factor (K_s) as calculated using stability functions approach for the sway frames. For non-sway frames column 6 and 7 shows K_f and K_s respectively. Table 4.9 shows λ_{cf} and λ_{cs} for sway frames in columns 2, and 3 respectively, and for non-sway frames they are in column 4, and 5.

For sway frames finite element approach shows good agreement with stability functions approach, comparing λ_{cs} with λ_{cf} , the maximum error was in frame 1 and its about 5%. Followed by the error in frame 3 (4%), and it's less than 1% for other frames. comparison between K_f and K_g also shows the same trend, that is, the accuracy of the finite element approach seems to be satisfactory for sway frames.

For no-sway frames the finite element approach shows disagreement with the results of the stability functions approach. The former gives higher values for λ_c and smaller values for k-factors. This is not a surprising result since the finite element approach is an approximate technique. This approximation comes from the assumption of the displacement functions which are imposed on the member. To clarify this point, a single column fixed at the base and hinged at the top is analysed by the two approaches, the stability functions approach gives as expected the exact value of K which is 0.699, while finite element approach gives K as 0.547. When the analysis is repeated and the column is divided into 10 elements K was founded to be 0.7. In the case when the restraint at the top removed the two approaches give K as 2.0. Finally a comparison of the CPU time for the two approaches is presented

Tables 4.1-4.8: Effective length factors for columns in braced and un-braced frame as calculated using alignment charts, finite element approach, and stability functions approach

Table (4-1)
Frame 1

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,3	3.13	3.21	3.13	.95	.766	.95

Table (4-2)
Frame 2

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,3	1.5	1.448	1.441	.67	.525	.662
2,4	1.94	2.047	2.038	.91	.743	.936

Table (4-3)
Frame 3

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,4	1.12	1.092	1.095	.62	.438	.602
2,5	1.25	1.337	1.342	.75	.536	.738
3,6	1.22	1.569	1.574	.74	.926	.866

Table (4-4)
Frame 5

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,3	2.0	2.016	2.019	.74	.586	.729
2	1.9	2.047	2.05	.7	.595	.74

Table (4-5)
Frame 4

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
21,31	1.3	1.215	1.212	.67	.476	.619
22,32	1.65	1.727	1.723	.86	.677	.88
23,33	1.69	1.835	1.832	.865	.719	.935
24,34	1.61	1.771	1.768	.85	.694	.903
25,35	1.61	1.92	1.916	.85	.752	.979
26,36	1.58	2.114	2.11	.85	.829	1.078
27,37	1.63	2.01	2.0	.86	.787	1.024
28,38	1.77	2.352	2.347	.88	.922	1.199
29,39	1.8	2.523	2.518	.88	.989	1.268
30,40	1.7	3.922	3.914	.87	1.537	1.999

Table (4-6)
Frame 6

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,3	1.13	1.125	1.127	.62	.456	.644
2	1.11	1.1	1.102	.605	.444	.629
4,12	1.25	1.294	1.296	.75	.524	.741
5,13	1.2	1.4	1.403	.725	.567	.802
6,14	1.15	1.484	1.487	.68	.601	.85
7,15	1.	1.964	1.967	.65	.795	1.124
8	1.18	1.108	1.11	.71	.447	.634
9	1.14	1.188	1.19	.665	.48	.679
10	1.1	1.1	1.102	.625	.444	.629
11	1.05	1.41	1.413	.58	.57	.807

Table (4-7)
Frame 7

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1,19	1.2	1.17	1.17	.69	.553	.636
2,20	1.47	1.609	1.61	.82	.761	.874
3,21	1.5	1.8	1.81	.83	.857	.983
4,22	1.42	2.111	2.11	.815	.998	1.145
5,23	1.36	2.118	2.12	.79	1.0	1.151
6,24	1.32	3.242	3.249	.77	1.534	1.76
7,13	1.19	1.095	1.097	.64	.517	.594
8,14	1.35	1.506	1.51	.79	.711	.817
9,15	1.31	1.367	1.37	.77	.645	.741
10,16	1.26	1.596	1.6	.75	.754	.867
11,17	1.2	1.519	1.52	.71	.718	.824
12,18	1.175	2.319	2.329	.7	1.095	1.258

Table (4-8)
Frame 8

Column No.	un-braced			braced		
	Kchart	K-F.E	K-S.F	Kchart	K-F.E	K-S.F
1	1.23	1.221	1.216	.65	.497	.683
2	1.2	.971	.964	.64	.419	.568
3,4	1.34	1.385	1.37	.67	.597	.806
5	1.34	1.35	1.33	.66	.617	.836
6	1.74	1.822	1.82	.87	.76	1.02
7	1.63	1.471	1.46	.85	.635	.861
8,9	2.08	2.09	2.08	.91	.903	1.226
10	2.06	2.078	2.05	.91	.93	1.26
11	1.76	1.89	1.97	.88	.845	1.13
12	1.65	1.624	1.613	.86	.7	.951
13,14	2.12	2.31	2.3	.92	.99	1.35
15	2.1	2.298	2.27	.915	1.02	1.39

Table (4-9)
Critical load factors

Frame	un-braced		braced	
	λ_{cf}	λ_{cs}	λ_{cf}	λ_{cs}
1	3.672	3.85	64.5	41.529
2	2.15	2.16	16.34	10.28
3	31.419	31.21	195.6	103.16
4	9.57	9.6	62.345	36.868
5	2.548	2.538	30.17	19.5
6	7.678	7.652	46.935	23.445
7	9.2	9.16	41.15	31.2
8	3.44	3.5	18.448	10.05

Table (4-10)
Comparison of CPU time

Frame	stability functions time(sec.)	finite element time(sec.)
1	1.62	.52
2	1.09	1.02
3	1.44	1.44
4	5.22	5.13
5	1.77	1.65
6	5.62	3.22
7	9.41	8.35
8	2.33	2.1

in table 4.10. As expected the finite element approach always requires less CPU time than the stability functions approach. However the differences are quite small.

4.1.2 Comparison with alignment charts :-

Charts are the most common procedure for finding the effective length factors. The charts were developed to be used for multistorey, multi-bay rigid frames and by making certain assumptions about the loading condition and buckled configuration, among these, the most important are :

- 1) All columns in the frame have the same stability parameter $\phi = \sqrt{P/EI}$. Actually ϕ will differ from column to column in the frame depending on the loading conditions.
- 2) All columns in the frame buckle simultaneously such that end rotation of the column under consideration are alternately equal to the column end rotation on all floors above and below.

In tables 4.1 - 4.8 columns 2 and 5 shows the effective length factors K_{chart} which are calculated by the charts for sway and no-sway frames, respectively. The relative stiffness ratio for the joint (G) has been taken as infinity for

columns with pinned supports, and as zero for columns with fix supports.

For frame 1, (the symmetrically loaded one storey one-bay frame which meets all the assumptions made in derivation of the alignment charts) the effective length ratios, from the alignment charts agree exactly with the computer solutions using stability functions in sway and non-sway cases.

For frame 2 (fig. 4.2 and table 4.2) K_{chart} is found to deviate from the exact results by -4% for columns 1 and 3 in the first storey, this means that the charts overestimates the effective length factors for the first storey. For columns in the top storey, error in K_{chart} is found to be about 5% or the charts underestimate effective length factor in this case. For the non-sway frames the same trend is also shown.

For the three storey-one bay frame (frame 3) shown in fig 4.3, table 4.3 shows that errors for columns in the first storey become less (2.3%) while they increase to about -23 % for the top storey. Errors in columns 2 and 5 in the second storey were about 7%. For the braced case errors for the

first and second storeys are less than 3%. While they are about -27% for the top storey.

, For frame 4 (fig 4.4 and table 4.5) which is a ten storeys-one bay tower, the minimum error was recorded in the first storey (about 4%), and it is maximum in columns at the top storey (about -56%). For the first four storeys errors were less than 9% and between 16% - 56% for the remaining storeys. For the non-sway frames, the minimum error was about -2% for columns in the second storey, the maximum error was also in the top storey (about 56%). For the first 4 storeys errors are less than 8% and they are between (13-56)% for the remaining.

For frame 5 (one storey-two bay frame shown in Fig 4.5 and Table 4.4) the maximum error were founded in column 2 and it is about 7%. For other columns errors was less than 1%. In the braced case the maximum error was less than 5%.

for frame 6 (Fig 4.6 and table 4.6) the largest error occurs in the external columns at each floor. it reaches 49% in the top storeys, for internal columns the error also increases from bottom to top but it is acceptable except for the top storey column (about 26%). The same is true for the

braced frame.

For frame 7 (fig 4.7, table 4.7) errors were minimum at the base and are maximum at the top for external and internal columns. Its not clear however, that internal columns consitently exhibit less error than external columns.

For fram 8 (fig 4.8 and table 4.8) the error was maximum in columns 2 and 7 (24% and 11% respectively). For other columns errors are larger in the top storey. The errors in Kchart for other storeys were small (less than 2%).

4.2 Elastic - plastic analysis :

The results of elastic-plastic analyses carried out by PLAS include the plastic collapse load factor, position of the plastic hinges (mechanism), and forces in members at λ_p . Table 4.11 shows λ_p for some selected frames from these shown in figures(4.1-4.8). These frames where analyzed for the loading shown in figures (4.9 - 4.10) but the uniformly distributed loads are replaced in each loaded member by three concentrated loads at the two ends and midspan. Figures 4.11, 4.12, 4.13 shows the collapse mechanism for some of these

frames. Discussion of the results and their relation to elastic stability analysis is made in the next chapter.

Table (4-11)
Collapse load factors

Frame no.	C.L.F
1	.30461
2	.27751
4	1.3331
7	3.3619
8	.18592

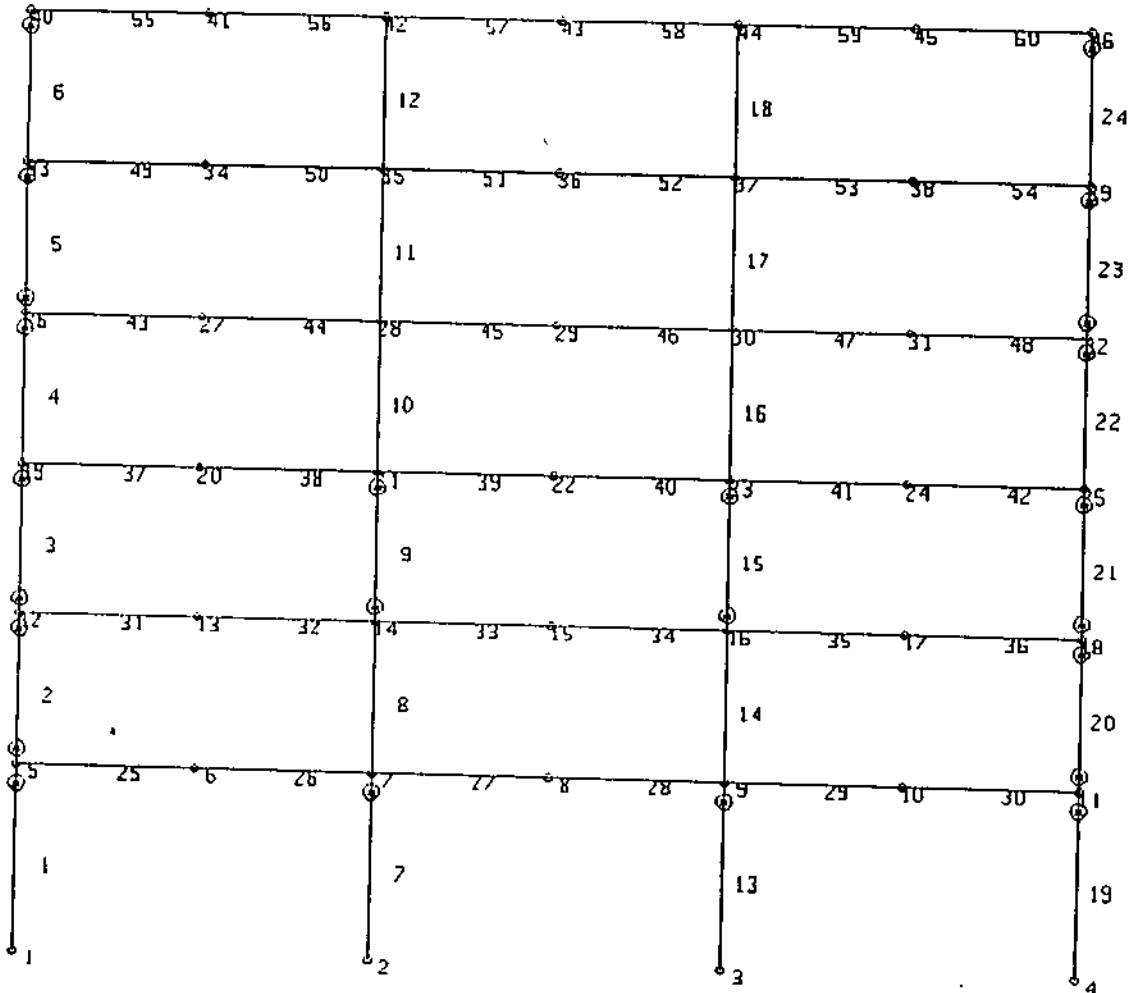


Fig.(4.11) Frame...7

Collapse load factor = 3.3891

CHAPTER IV

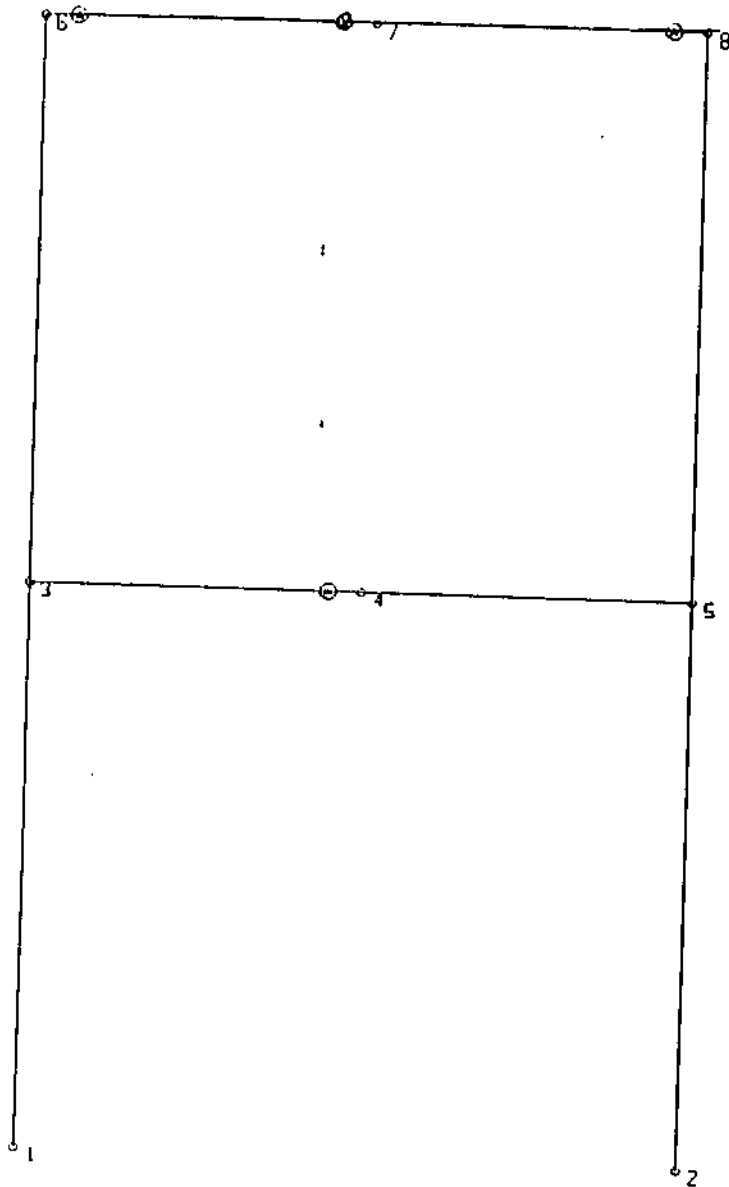


Fig.(4.12) Frame...2

Collapse load factor = .27751

CHAPTER IV

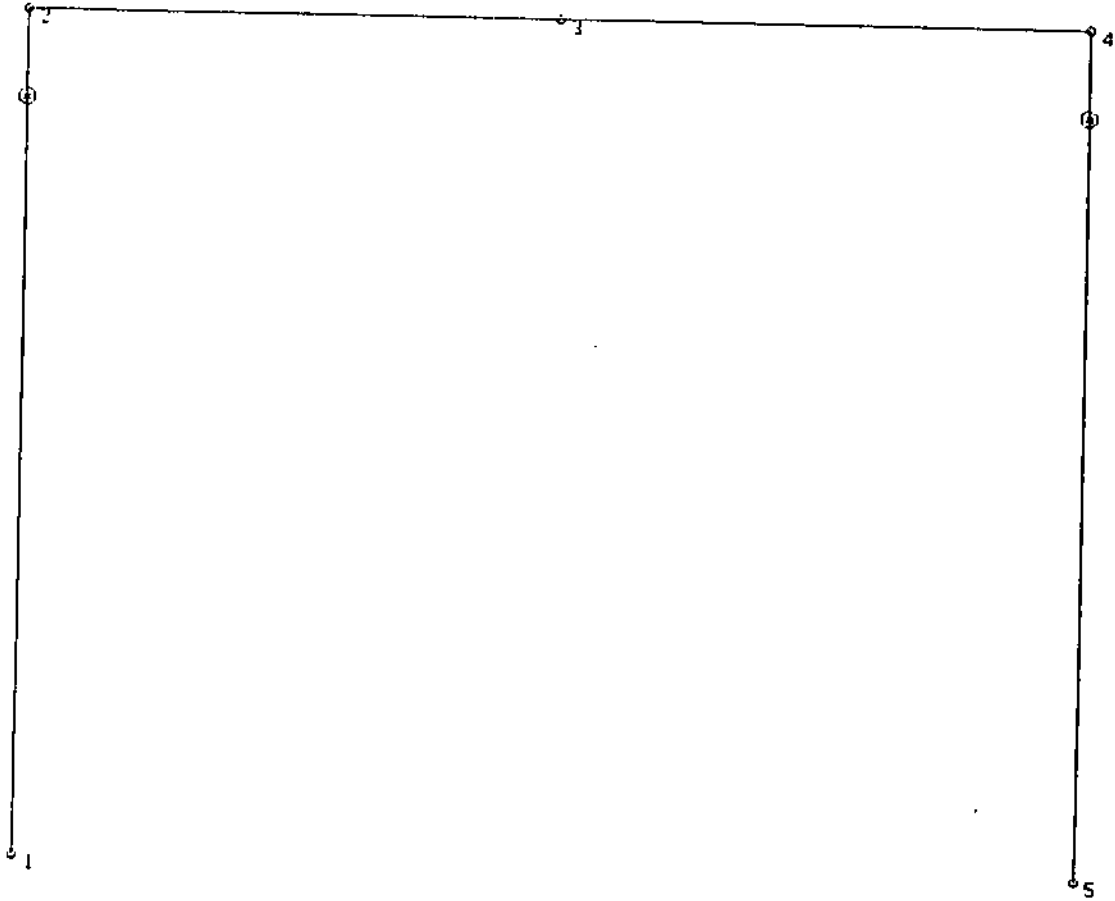


Fig.(4.13) Frame...1

Collapse load factor = .30461

CHAPTER V

DISCUSSION

The aim of this chapter is to discuss the results previously presented in chapter four, and the basic principles behind them. The chapter is divided into two sections, the first part deals with the results of the elastic stability analysis and the other with those of the plastic analysis.

5.1 ELASTIC STABILITY ANALYSIS:

The main objective of elastic stability analysis, as it was shown in chapter two, is to find critical load factor λ_c .

From a practical point of view, λ_c has limited use in the design procedures as set by current codes of practice. Instead each member is considered separately using the effective length factor concept to account for the interaction of the member with elements connected to its end on the one hand and to other members at its storey level on

the other hand.

5.1.1 EFFECTIVE LENGTH FACTOR-DEFINITION AND USE

ACCORDING TO AISC CODE:

In principle, Euler elastic buckling governs the capacity of members with large slenderness ratios, yield stress F_y applies for short columns, and a transitional state prevails in between.

The slenderness ratio is defined as KL/r where L and r are length of the column, and radius of gyration respectively. K is the effective length factor which is equal to one for a pinned ended non swaying member and takes other values for different end restraints that tend to reflect the shape of buckling of the member. KL may be viewed as the distance between two inflection points, either real or imaginary on the buckled curve of the member.

Actually, the K -factor is well defined only for idealized cases. For other cases, (i.e., frame column) it is difficult to evaluate the degree of end restraint offered by members that frame into a given column and, indeed the full

interaction of all the members of a steel frame. Thus an analysis is necessary to determine the effective length factor K . One acceptable "rational" approach, adapted by AISC, is to use the alignment charts.

Once the K -factor is calculated, the design process for a column can be carried out by calculating the allowable stress F_a , and then comparing it with the applied stress f_a . F_a is, of course, dependent on KL/r .

5.1.2 EFFECTIVE LENGTH FACTOR AS CALCULATED BY ESAP

The output of ESAP includes the elastic critical load factor λ_c for the frame, and member forces at this load factor. Effective length factors for compression members are then calculated by correlating the force in the member at the stage of buckling of the structure with the Euler buckling force of the member. This is shown in Eq.(4.1). In this procedure the user needs not to specify the end restraints for each member as this is accounted for automatically by the complete stability analysis of the structure. The KL/r value obtained here reflects more accurately the slenderness of a member in the frame for a particular pattern of loading.

If only one member with well defined boundary conditions constitutes the structure, then the effective length as obtained by ESAP exactly matches values obtained theoretically. Furthermore, if a simple frame, say a portal frame, which satisfies all the assumptions made in the derivation of the alignment charts, is analyzed using ESAP, then the program will yield identical results to those calculated using the alignment charts.

EFFECT OF MEMBER FORCES (PRIMARY BENDING MOMENTS) ON λ_c
AND K-FACTORS:

As stated in chapter three ESAP can deal with member forces either uniformly distributed over the full span or concentrated at midspan. The frames in chapter four were analyzed assuming vertical loads to be lumped at the ends of the member. For the purpose of isolating the influence of the distribution of the load on stability, the frames in Fig. 4.1-4.8 are analyzed using the lumped load and the distributed load.

The results of the analyses are shown in table 5.1 where

it is clear that for the frames considered, the primary bending moment has little effect on λ_c of the structure. This also applies to the k-factors.

EFFECT OF LATERAL LOADS (WIND LOADS):

After the frames (2, 3, 4, 6, 7 & 8) were analyzed subjected to vertical loads (dead and live loads) lateral loads were added. The lateral loads are wind loads and they are shown in Fig.(5.1).

Results of the analyses show that as far as λ_c is concerned (table 5.2) the effect of the lateral loads is almost negligible. That is to say that a frame buckles under nearly the same intensity of vertical load of another identical frame which is subjected to lateral loads on top of these vertical loads.

The cases considered only involved wind loads based on the Jordan code for wind speed of 35 m/s. For earthquake loads or for wind forces for areas where higher wind speed and hence pressure is expected, the effect of lateral loads on λ_c would be more prominent. Some examples were carried

out which showed that the difference is still reasonably small.

Table (5-1)

Effect of primary bending moment on λ_c

Frame	critical load factor λ_c	
	U.D.L	Joint loads
2	2.1599	2.16
3	31.21	31.19
4	9.304	9.6
5	2.532	2.538
6	7.652	7.657
7	9.15	9.16
8	3.5	3.47

Table (5-2)

Effect of lateral loads on λ_c

Frame	Critical load factor λ_c	
	Vertical loads	Combined loads
2	2.1599	2.5
3	31.21	30.84
4	9.304	9.42
6	7.652	7.535
7	9.15	9.08
8	3.5	3.5

Table (5-3)

Effect of lateral loads on effective length factors for frame 8

Column No.	Effective length factor		
	Kchart	Vertical loads	Combined loads
1	1.23	1.216	1.074
2	1.2	.964	.977
3	1.34	1.37	1.317
4	1.34	1.37	1.388
5	1.34	1.35	1.449
6	1.74	1.82	1.651
7	1.63	1.46	1.477
8	2.08	2.08	2
9	2.08	2.08	2.09
10	2.06	2.05	2.167
11	1.76	1.97	1.8
12	1.65	1.613	1.64
13	2.12	2.3	2.21
14	2.12	2.27	2.33
15	2.1	2.27	2.34

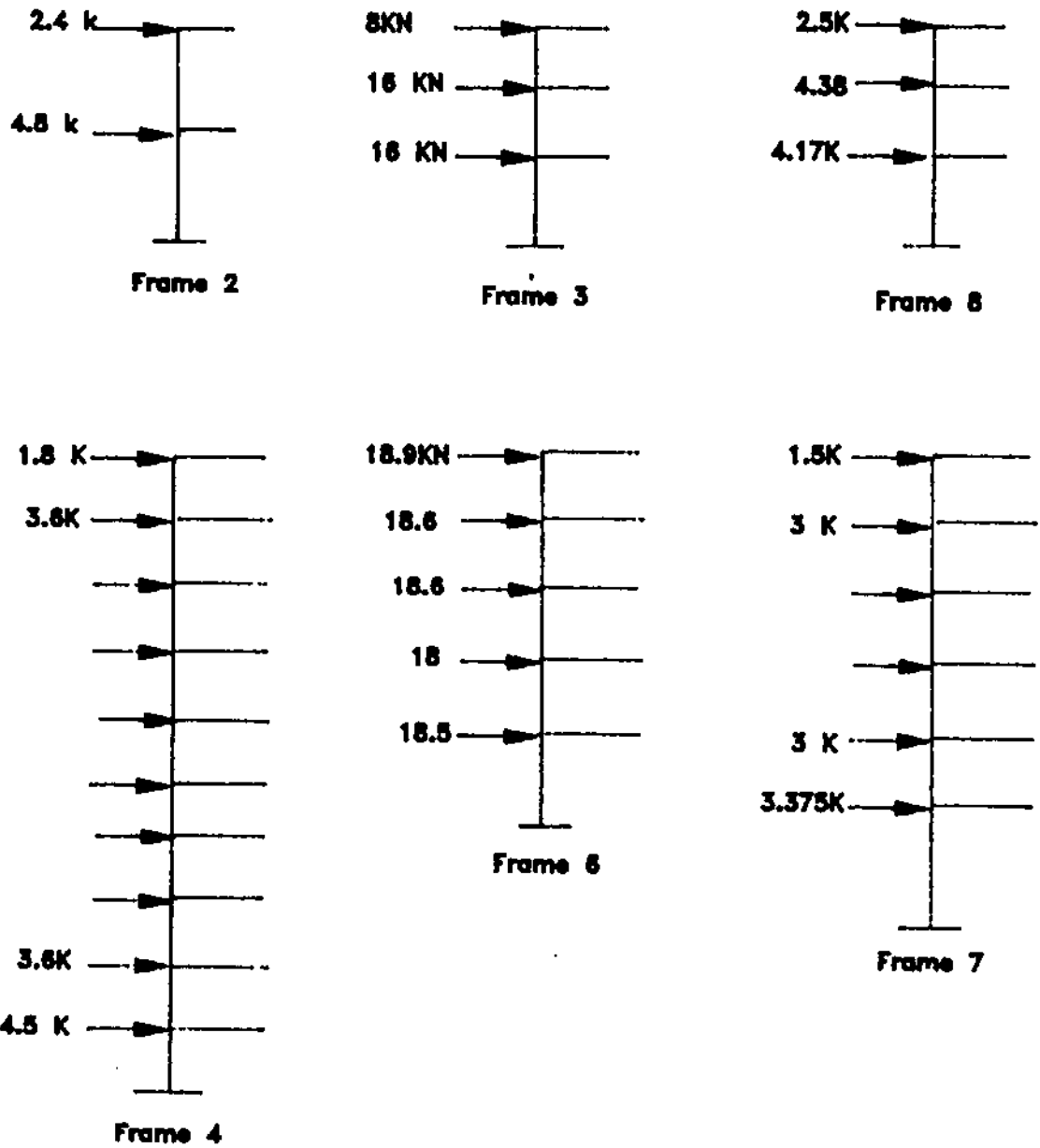


Fig.(5.1) Lateral loading (wind load)

The effect of the wind loads on the K-factors is more visible. This is shown for frame eight in table (4.3). K-factors for external columns in the side subjected to wind loads increase while it decrease for columns on the other side, this is expected as wind loads decrease the compression in the windward columns and increase it in the columns in the opposite end.

5.1.3 DISCUSSION OF K-FACTORS RESULTS:

Tables 4.1 - 4.8 show K-factors for compression members as calculated by alignment charts, and computer, once by the finite element approach and another time using stability functions approach, for sway and non sway frame.

According to the tables only for frame one, exact agreement were found between alignment chart and the stability functions computer solution. This is because frame one is the only frame considered that satisfies all the assumptions intierent in the alignment charts.

Tables 5.4-5.6: Errors resulting from using alignment charts to calculate effective length and stress ratio

Table (5-4)

Frame 2A

$KL/r=42.7$ for columns 1,3 and 59 for columns 2,4

Column No.	Effective length			stress ratio		
	Kchart	Kcom.	differance%	chart	com.	differance%
1,3(W12X40)	1.35	1.33	-1.5	.454	.453	-0.22
2,4(W12X40)	1.55	1.84	15.8	.754	.762	1

Table (5-5)

Frame 2B

$KL/r=82.45$ for columns 1,3 and 117 for columns 2,4

Column No.	Effective length			stress ratio		
	Kchart	Kcom.	differance%	chart	com.	differance%
1,3(W14X61)	1.08	1.05	-2.8	.539	.533	1
2,4(W14X61)	1.12	1.486	25	.797	.829	4

Table (5-6)

Frame 7A

Column No.	Effective length			stress ratio		
	Kchart	Kcom.	differance%	chart	com.	differance%
1,19(W14X90)	1.12	1.141	1.8	.862	.864	.2
2,20(W14X90)	1.4	1.569	10.77	.885	.9	1.6
3,21(W14X74)	1.35	1.656	18.5	.972	.99	1.8
4,22(W14X74)	1.32	1.93	31.6	.907	.927	2.15
5,23(W14X61)	1.3	2.057	37	.846	.852	.7
6,24(W14X61)	1.28	3.134	68	.95	.98	3
7,13(W14X132)	1.17	1.075	-9	.957	.947	-1
8,14(W14X132)	1.3	1.478	12	.799	.812	1.6
9,15(W14X82)	1.22	1.346	9.4	.988	.99	.2
10,16(W14X82)	1.26	1.574	20.3	.733	.755	3
11,17(W14X38)	1.11	1.293	14	.975	.991	1.5
12,18(W14X38)	1.1	1.977	44	.484	.529	8.5

For other frames the alignment charts either overestimate or underestimate K . For example, it is clear that for frame 2 the charts overestimate and underestimate K -factors in the first and second storeys respectively, but with acceptable ratios (-4% and 5%). For the three-storey one bay frame (frame 3) acceptable agreement between alignment charts and computer solutions prevails in the first and second storey, yet a larger error is found in the third storey. Simillar results was found in frame 4 as errors for the first four storeys are less than 10% but this increses to about 56% in uper storeys. Actually this trend is expected due to the assumptions used in developing the charts, esspecially that which assumes the stability parameter $\phi = \sqrt{p/EI}$ to be equal for all the columns in the frame and that all the columns buckle simultaneously.

In the other frames the maximum error is also found in the top storeys except in frame 8 where it occurs in column 2 in the first storey while the error found in column 12 at the top storey was only 2%. This is because column 2 has a considerably different I value than 3 and 4, yet the value of P is not expected to be that different for those three

columns. This necessarily will result in some error when the alignment charts are used.

To summarize, it is true that using the alignment charts produces serious errors in calculating K-factors and that these errors will affect the design of the frames columns. Furthermore these errors are not consistently conservative or otherwise.

For the purpose of estimating the influence of these errors in design, frames 2 and 7 are designed according to ASIC code using "STAAD3" 'structural analysis and design program' for the loading shown in Figs. 4.9 and 4.10, STAAD3 enables the user to input K-factors for the members so the frames are design using k-factors as calculated by the charts and computer. The final steel sections, the percentage difference between K_{chart} and K_{com} and finally the stress ratios are shown in tables 5.4, 5.5 and 5.6. The stress ratio (based on AISC) is the combined bending and axial stress ratios for which the member is designed and it should always be less than 1.

For frame 2 using K_{chart} and K_{com} yielded the same steel

sections and the maximum difference in stress ratio was less than 1% while the maximum error in the K-factors was 15.8%. In other words, errors in K were not reflected in design. This is because :

(1) Columns are design about major axis (for low KL/r) where errors in K produce minor difference in the allowable stress.

(2) The maximum errors in K-factors were found in columns 2 and 4 in the top floor. In these columns the bending stress is high compared to the axial compression stress and it is known that the bending stress is less affected by slenderness ratio. When frame 2 columns are designed about minor axis (high KL/r) the maximum difference in stress ratio increased to about 4%. Actually the difference between stress ratios increased for all columns in the frame and also the errors in K-factors.

For frame 7 (table 5.6) the maximum difference in stress ratio is 8.5% for column 12 followed by 3% for column 6, while the error in the K-factor is maximum for column 6 (68%) followed by column 12 (44%). Columns 6 and 12 are in the top storey where 6 is an external column and 12 is an internal one. Working axial compression stress (f_a) for

column 6 is 2.32 Ksi, and its axial compression allowable stress using K_{com} (Fa_{com}) is 15.85 while it's 19.88 using K_{chart} . Hence $fa/Fa_{com} = 0.14$ and $fa/Fa_{chart} = 0.12$.

In both cases $fa/Fa < 0.15$ which requires the use of the AISC formula (1.6-2) where K-factor does not affect the bending stress and the error from compression stress is small since the slenderness ratio for this column is low. Even if KL/r is increased further, the error would not be serious as the contribution of the axial compression stress to the total stress is small. In other words, these columns behaved almost as beams, and because of this the errors in K-factors obtained from the charts is large.

For column 12, the errors in the stress ratio are the largest. fa for this column is 8.33, and Fa_{com} and Fa_{chart} are 18.47 and 20.14 respectively, so

$$fa/Fa_{com} = 0.45 \quad \text{and} \quad fa/Fa_{chart} = 0.4137$$

the stress ratio for this column is 0.529 which means that the contribution of bending stress here is small and the column is not heavily loaded as it was grouped with the lower

columns as is customary in design. For this member it is clear that most of the errors in stress ratio come from errors in axial compression stresses which are relatively high.

It is observed that small error in the stress ratio occurs when the stress ratio is close to 1. This condition indicates that either the member dominates the instability of the frame or else most of its stress comes from bending and both these situations bring the alignment charts procedure results close to those obtained by the computer analysis. It is worth mentioning also that the condition of stress ratio close to 1 is the most important condition for design purposes.

5.2 ELASTIC PLASTIC ANALYSIS:

Elastic plastic analyses for some selected frames are carried out using PLAS. Results of analyses are shown in table (5.7) such that the load factors based on stability analysis once and on plastic analysis another time are normalized to produce collapse load factor equal to two.

As mentioned earlier effects of axial forces on plastic

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As mentioned earlier effects of axial forces on plastic

moments and member stiffness were included in PLAS, so the collapse load factors λ_f shown in table (5.7) can be considered as the failure load for the frames, while λ_p is the plastic collapse load factor considering the effect of axial forces only on the plastic moment of the sections. λ_{MR} is the failure load calculated using the Merchant-Rankine formula which is given as

$$1/\lambda_{MR} = 1/\lambda_p + 1/\lambda_c$$

As shown by table (5.7) λ_{MR} is less than λ_f except for frame 8 where the ratio $\lambda_c/\lambda_p = 15.5$ which is greater than 10, Merchant-Rankine formula is recommended for $4 \leq \lambda_c/\lambda_p \leq 10$.

In addition to the frames shown in table(5.7) two other frames are analyzed using PLAS, these frames are 2A, 7A and they are designed according to the working loads, the failure loads for these frames are 2.84 and 2.9 respectively.

Table (5-7)
Comparison between failure load factor
calculated using PLAS and failure load
factor from Marchent-Rankine Formula

Frame No.	λ_f	λ_p	λ_c	λ_{MR}
1	2	2.002	25.279	1.85
2	2	2.003	15.567	1.77
4	2	2.03	14.66	1.789
7	2	2.08	16.428	1.84
8	2	2.42	37.65	2.26

CHAPTER VI

SUMMARY AND CONCLUSION

In this work two computer programs have been developed, they are :

- (1)ESAP "Elastic stability analysis program" which is used to find the elastic critical load factor λ_c for plane frames, and then to calculate the effective length factors for compression members in the frame, using the finite element approach and the stability functions approach.
- (2)PLAS or elastic-plastic analysis program which is used to find the failure load factor for a plane frame and member forces at this stage.

The programs are then used to analyze some selected frames which are shown in Fig's 4.1-4.8. From analyses results the following conclusions can be drawn:

- (1)Comparison between finite element approach and stability functions approach used in the elastic stability analysis shows that there is a good agreement between the two approaches when unbraced frames are analyzed. For braced frames, the finite

element approach fails to predict λ_c accurately. With respect to CPU time, finite element approach always requires less time than stability function approach, but the difference is quite small.

- (2) A comparison of K-factors from computerized analysis with those from alignment charts shows that, only for frame 1 (one story, one bay frame) the alignment charts show exact agreement with the computer solution. For other types of frames the effective length ratios as determined from the charts, can be considerably in error, either conservatively or unconservatively depending on the frame and loading.
- (3) The influence of errors in effective length on design as determined by the criteria of allowable stress is found to be quite small.
- (4) The presence of the primary bending moments did not affect the stability of the considered frames.
- (5) The effect of lateral loads on λ_c is found to be small while its effect in K-factors is more significant.

APPENDIX I DATA ENTRY

The data input for the programs are free format, where the individual pieces of data are separated by commas. The order in which the data should be input is as follows:-

- Data about number of joints ,number of elements, number of material properties, and number of restrained joints are inputed in the first line. For PLAS the yild stress of steel is added.
- Data about joint coordinates, where number of the joint, nstep, X- coordinate and Y-coordinate are inputed in the same line. If data generation is used then nstep should indicate number of steps to reach from the previous joint to the current one.
- Data regarding the nodes number at the ends of the elements, for each element, number of the element,nstep,number of node at end I,and number of node at end J, should be input in one line. In the case where data generation is used nstep should be grater than one and only data for basic elements are inputed.
- Data regarding the material group number to which the

element belong, in each line of data number of the element, nstep, and the group number are inputted in one line.

- Data about the material and cross-sectional properties of the material groups of the elements, for each group, area, inertia, modules of elasticity, are input in one line. For PLAS plastic moment of the section should be added.
- Data regarding the restrained joints, for each restrained joint, the number of the joint, restrained in x direction, y direction and rotation are input in one line. The joint is considered restrained in a special direction if 1 is input for this direction, else 0 should be input.
- Data regarding the loads on the structure, here data about number of loaded joints and members are input in one line then data about loaded joint are inputted where for each loaded joint, number of the joint followed by its loading in X, Y, and rotation are inputted in the same line. After this data about loaded members are inputted, where for each loaded member, number of the member, uniformly distributed load in X-direction (UDL_x), UDL_y, mid-span load in X-direction, and mid-span load in Y-direction are inputted in one line. For PLAS only data about loaded

joints are excepted.

- In the last line increment by which the initial load factor is increased and the accuracy, which is an indication to stop the program when the difference between two successive load factor is less than this accuracy should be inputted. This line is special for ESAP.

As example of data files the data file for frame 8 is presented here.

DATA FILE FOR FRAME 8

20,27,5,5	NPION,NELEN,NMATS,NROJ
1,1,0,0-----	
17,4,1440,0	
2,1,0,220	
18,4,1440,220	JOINT COORDINATES
3,1,0,400	
19,4,1440,400	
4,1,0,640	
20,4,1440,640-----	
1,1,1,2.....	
5,4,17,18	
6,1,2,3	
10,4,18,19	
11,1,3,4	
15,4,19,20	MEMBERS ENDS NUMBERING
16,1,2,6	
19,3,14,18	
20,1,3,7	
23,3,15,19	
24,1,4,8	
27,3,16,20.....	
1,1,1-----	
11,2,1	
2,1,2	
12,2,2	
3,1,3	
13,2,3	
4,1,3	MEMBERS MATERIALS GROUPS
14,2,3	
5,1,2	
15,2,2	
16,1,4	
19,3,4	
20,1,5	
27,7,5-----	
26.5,362,29000	
38.8,548,29000	
29.1,1110,29000	MATERIAL GROUPS
14.7,800,29000	
7.68,301,29000.....	
1,1,1,1-----	
5,1,1,1	
9,1,1,1	RESTRAINED JOINTS
13,1,1,1	
17,1,1,1-----	
3,12<-----	NO. OF LOADED JOINTS AND MEM.
2,4.167,0,0.....	
3,4.375,0,0	LOADED JOINTS
4,2.5,0,0	

16,0,-.89,0,0----->
 17,0,-1.05,0,0
 18,0,-1.05,0,0
 19,0,-.89,0,0
 20,0,-.89,0,0
 21,0,-1.05,0,0 LOADED MEMBERS
 22,0,-1.05,0,0
 23,0,-.89,0,0
 24,0,-.78,0,0
 25,0,-.89,0,0
 26,0,-.89,0,0
 27,0,-.78,0,0----->
 5,.001----->LOAD FACTOR AND ACCURACY

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ملخص

لقد تم تطوير برنامج حاسوب لدراسة الاستقرار العام للمنشآت الهيكلية بهدف دراسة ومقارنة طريقة العناصر المحدودة وطريقة دالات الاستقرار ثم مقارنة نتائج هذه الطرق بخصوص الطول الفعال مع طريقة المخططات البيانية . ودرس أيضاً تأثير الأخطاء عند حساب الطول الفعال بواسطة المخططات البيانية على عملية التصميم .

وتم أيضاً دراسة تأثير عزم الانحناء الرئيسي والقوى الأفقية على استقرار المنشآت من ناحية الحمل الاستقراري الحرج والأطوال الفاعلة .

أخيراً تم تطوير برنامج آخر للتحليل بفرضية المرونة واللدونة المثلى بهدف حساب حمل الانهيار للمنشآت وملاحظة علاقته مع الحمل الاستقراري الحرج المحسوب بواسطة البرنامج الأول .